

# PARAFAC RECEIVER FOR BLIND MULTIUSER EQUALIZATION IN WIRELESS COMMUNICATION SYSTEMS WITH TEMPORAL OVERSAMPLING

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## ABSTRACT

In this paper we present a new blind receiver for multiuser signal separation and equalization. The proposed receiver is designed for wireless communication systems employing multiple antennas and temporal oversampling at the receiver and relies on parallel factor (PARAFAC) analysis. Based on a parametric multipath channel model with frequency-selective fading, we perform a PARAFAC decomposition of the 3-D received signal and present a blind receiver algorithm for multiuser equalization. The proposed receiver algorithm is composed of two processing steps. First, co-channel user signals are separated in the 3-D (space  $\times$  time  $\times$  oversampling)-domain using an alternating least squares (ALS) procedure. Then, users' sequences are individually equalized in the time-domain using a subspace algorithm. Simulation results compare the performance of the proposed receiver with that of classical ones.

## 1. INTRODUCTION

The development of advanced signal processing techniques for wireless communications is an attractive research topic. In multiuser (mobile) wireless communication systems, the main task of receiver signal processing is to identify the parameters of the propagation channel and/or to recover the useful transmitted information in the presence of co-channel interference, intersymbol interference and additive noise. One challenging research topic in the context of wireless communications is that of blind multiuser equalization, which consists in recovering the transmitted information of several co-channel users with the assumption of a frequency-selective channel and without the knowledge of training sequences.

Working on a two-dimensional (2-D) space-time model, one can resort to several existing blind array processing algorithms that simultaneously exploit both space and time dimensions of the received signal in order to recover the transmitted signal of all users [1]. These algorithms may exploit either the algebraic structure or the statistical properties of received signal matrix. Blind algorithms generally take into account special (problem-specific) structural properties of the transmitted signals (e.g. orthogonality, finite-alphabet, constant-modulus or cyclostationarity) for multiuser equalization [2, 3]. Without these additional considerations, it is well known that the low-rank property of matrices is not enough to guarantee a unique model for the received signal. This lack of inherent uniqueness is one of the limitations of a 2-D modelling for the received signal in wireless communication systems.

Unlike the decompositions of 2-D arrays (matrices), which is generally nonunique for any rank greater than one (for rank one it is unique up to a scalar factor), the decomposition of 3-D arrays (also called third-order *tensors*) can be unique up to a scalar factor for low-enough ranks [4]. One of the most studied low-rank decompositions of 3-D (or higher dimensional) tensors is called PARAFAC (parallel factor) analysis, which was developed by Carroll and Chang [5] and Harshman [6] as a data analysis tool in psychometrics. It has also been widely studied in the context of chemometrics [7]. In the context of wireless communications, PARAFAC has recently appeared as a powerful tool from a receiver signal processing perspective, allowing us to identify channel parameters and to recover user symbols without imperatively utilizing structural properties/constraints. It is also worth noting that a 3-D (tensorial) model for the received signal results from an additional "axis" or dimension in the received signal model instead of the usually considered *space* and *time* dimensions. In wireless communications, this means that diversity can also be exploited in this additional third dimension. Most of research bringing PARAFAC to the context of signal processing for wireless communications were carried out by Sidiropoulos and his co-workers (see [8] and references therein).

In this work, we present a PARAFAC receiver for blind multiuser equalization. We consider a single-input multiple-output (SIMO) wireless communication system with a receiver antenna array, where the received signal at each antenna output is oversampled in the time-domain. The proposed receiver exploits the 3-D structure of the received signal, which is organized as a third-order tensor with space, time and oversampling dimensions. Based on a parametric frequency-selective multipath channel model, a PARAFAC model for the received signal is developed and exploited for multiuser signal separation. The blind receiver is divided in two sections. First, co-channel user signals are separated in the 3-D (space  $\times$  time  $\times$  oversampling) domain using an alternating least squares (ALS) procedure [7]. Then, users' sequences are individually equalized in the time-domain using a subspace algorithm [9]. Simulation results are provided to compare the performance of the proposed receiver with that of classical ones.

This paper is organized as follows. In Section II the multipath channel model is presented. In Section III a brief overview of PARAFAC analysis is given. In Section IV we develop the proposed PARAFAC receiver for blind multiuser equalization. Section V contains simulation results and Section VI concludes this paper.

## 2. CHANNEL MODEL

Let us consider a multipath wireless propagation environment where  $Q$  transmitted co-channel user signals are received by  $M$  antennas. The overall propagation channel is assumed time-dispersive (frequency-selective channel). In absence of noise, the discrete-time base-band representation of the signal received at the  $m$ -th antenna of a linear and uniformly-spaced array at the  $n$ -th time instant is given by

$$x_m(n) = \sum_{q=1}^Q \sum_{l=1}^L a_m(l) g(k-l) s_q(n-k), \quad (1)$$

where  $L$  denotes the number of paths of each user and  $a_m(l)$  is the fading envelope of the  $l$ -th path of the  $q$ -th user. In this model we have assumed that all users have the same number of multipaths in order to simplify mathematical notation. The term  $a_m(l)$  is the response of the  $m$ -th antenna-element to the  $l$ -th path of the  $q$ -th user,  $l$  being the angle of incidence or direction of arrival (DOA). Similarly, the term  $g(k-l)$  denotes the propagation delay (in multiples of the symbol period  $T$ ) and the term  $g(k-l)$  represents the  $k$ -th component of the pulse-shaping filter response. The length of the channel impulse response  $K$  is such that  $K \geq \max(l)$ . This condition guarantees that all multipath energy is captured in our frequency-selective channel impulse response model. Finally,  $s_q(n)$  is the symbol transmitted by the  $q$ -th user at the  $n$ -th time instant. Using vector notation, the  $n$ -th received signal after noise addition can be represented as

$$\mathbf{y}(n) = \mathbf{x}(n) + \mathbf{v}(n) = \sum_{q=1}^Q \sum_{l=1}^L \mathbf{a}_{lq} \mathbf{g}_{lq}^T \mathbf{s}_q(n) + \mathbf{v}(n), \quad (2)$$

where  $\mathbf{a}_{lq} = [a_1(l) \cdots a_M(l)]^T$ ,  $\mathbf{g}_{lq} = [g(-l) \cdots g(K-1-l)]^T$  and  $\mathbf{s}_q(n) = [s_q(n) \cdots s_q(n-K+1)]^T$ , with  $q = 1, \dots, Q$  and  $l = 1, \dots, L$ . Depending on the type of signal processing that we intend to use at the receiver, we may utilize either the above parametric channel model with explicit description of angles and delays (narrowband assumption) or a non-parametric model, when we are not interested in characterizing angle and delay parameters of the channel. This is generally the model adopted when the antennas undergo independent fading (diversity assumption). In this work we focus on the parametric model, which means that all the multipath parameters of all users are captured in our tensorial models.

In order to represent the received signal in a compact model, let us assemble the space and time responses of the  $L$  multipaths from the  $Q$  users into equivalent matrices. By defining  $\mathbf{b}_q = [1 \cdots L]^T$ ,  $\mathbf{A}_q = [\mathbf{a}_{1q} \cdots \mathbf{a}_{Lq}]$ ,  $\mathbf{B}_q = \text{Diag}(\mathbf{b}_q^T)$  and  $\mathbf{G}_q = [\mathbf{g}_{1q} \cdots \mathbf{g}_{Lq}]$  and collecting  $N$  consecutive samples of the received signal, the latter can be written as an  $M \times N$  matrix  $\mathbf{Y} = \mathbf{A} \mathbf{B} \mathbf{G}^T \mathbf{S} + \mathbf{V}$ , where  $\mathbf{A} = [\mathbf{A}_1 \cdots \mathbf{A}_Q]$  is an  $M \times LQ$  matrix containing spatial signatures,  $\mathbf{B} = \text{Diag}([\mathbf{b}_1^T \cdots \mathbf{b}_Q^T])$  is an  $LQ \times LQ$  matrix containing path gains,  $\mathbf{G} = \text{BlockDiag}(\mathbf{G}_1 \cdots \mathbf{G}_Q)$  is a  $KQ \times LQ$  matrix collecting the temporal signatures and  $\mathbf{S} = [\mathbf{S}_1^T \cdots \mathbf{S}_Q^T]^T$  is a  $KQ \times N$  matrix of symbols, with  $\mathbf{S}_q$  being a Toeplitz matrix characterized by its first row and column equal to  $\mathbf{s}_q^{(r)} = [s_q(1) s_q(2) \cdots s_q(N)]$  and  $\mathbf{s}_q^{(c)} = [s_q(1) 0 \cdots 0]^T$ , respectively. The operator  $\text{Diag}(\cdot)$  forms a diagonal matrix from its row

vector argument while the operator  $\text{BlockDiag}(\mathbf{G}_1 \cdots \mathbf{G}_Q)$  forms a block-diagonal matrix where the diagonal blocks are given by its argument matrices.

## 3. PARALLEL FACTOR (PARAFAC) ANALYSIS

For an  $I \times J \times K$  third-order tensor  $\mathcal{X}$ , its R-component PARAFAC decomposition is given by

$$x_{i,j,k} = \sum_{r=1}^R a_{i,r} b_{j,r} c_{k,r}. \quad (3)$$

The standard PARAFAC model for a three-way (3-D) array expresses the original tensor as a sum of rank-one three-way factors, each one of which being an outer product of three vectors. By analogy with the definition of matrix rank, the rank of a third-order tensor is defined as the minimum number of rank-one three-way components needed to decompose  $\mathcal{X}$ . The fundamental difference when going from matrices to tensors are their uniqueness. Rank- $R$  PARAFAC decompositions are essentially unique for a great range of  $R > 1$  [4].

The PARAFAC decomposition can also be represented in matrix notation. Define an  $I \times R$  matrix  $\mathbf{A}$ ,  $J \times R$  matrix  $\mathbf{B}$  and  $K \times R$  matrix  $\mathbf{C}$ . Define also a set of  $J \times K$  matrices  $\mathbf{X}_{i..}$ ,  $i = 1, \dots, I$ , a set of  $K \times I$  matrices  $\mathbf{X}_{.j.}$ ,  $j = 1, \dots, J$  and a set of  $I \times J$  matrices  $\mathbf{X}_{..k}$ ,  $k = 1, \dots, K$ . Based on these definitions, the model (3) can be written in three different ways. For each writing of the model a system of simultaneous matrix equations exists. The three writings of the model are:

$$\begin{aligned} \mathbf{X}_{i..} &= \mathbf{B} \mathbf{D}_i [\mathbf{A}] \mathbf{C}^T \quad i = 1, \dots, I \\ \mathbf{X}_{.j.} &= \mathbf{C} \mathbf{D}_j [\mathbf{B}] \mathbf{A}^T \quad j = 1, \dots, J \\ \mathbf{X}_{..k} &= \mathbf{A} \mathbf{D}_k [\mathbf{C}] \mathbf{B}^T \quad k = 1, \dots, K, \end{aligned} \quad (4)$$

where the operator  $\mathbf{D}_i[\mathbf{A}]$  forms a diagonal matrix from the  $i$ -th row of  $\mathbf{A}$ . The matrices  $\mathbf{X}_{i..}$ ,  $i = 1, \dots, I$ ,  $\mathbf{X}_{.j.}$ ,  $j = 1, \dots, J$ , and  $\mathbf{X}_{..k}$ ,  $k = 1, \dots, K$  can be interpreted as slices of the tensor along the first, second and third dimensions, respectively. Stacking these matrix slices columnwise into  $JI \times K$ ,  $KJ \times I$  and  $IK \times J$  matrices  $\mathbf{X}_1$ ,  $\mathbf{X}_2$  and  $\mathbf{X}_3$  we have

$$\mathbf{X}_1 = (\mathbf{A} \diamond \mathbf{B}) \mathbf{C}^T, \quad \mathbf{X}_2 = (\mathbf{B} \diamond \mathbf{C}) \mathbf{A}^T, \quad \mathbf{X}_3 = (\mathbf{C} \diamond \mathbf{A}) \mathbf{B}^T, \quad (5)$$

where  $\diamond$  is the Khatri-Rao (columnwise Kronecker) product.

Uniqueness of the PARAFAC decomposition was studied by Harshman [6] and the proof was provided by Kruskal [4]. According to Kruskal, a trilinear PARAFAC decomposition over  $\mathbb{R}$  is unique (except for the trivial permutation and scaling ambiguity) if  $k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2(R+1)$ , where  $k_{\mathbf{A}}$  is the  $k$ -rank of  $\mathbf{A}$ , defined as the maximum  $r$  such that every  $r$  columns of  $\mathbf{A}$  are linearly independent.

## 4. PARAFAC RECEIVER DESIGN

The proposed PARAFAC receiver for multiuser equalization is designed for wireless communication systems with joint use of multiple antennas and temporal oversampling at the receiver. Since the receiver is based on a 3-D PARAFAC model for the received signal, we start from (2) and take into account the temporal oversampling at each antenna output.

#### 4.1 Signal Modeling

By defining  $P$  as the oversampling factor, the temporal signature vector (i.e., pulse-shaping filter response) can now be represented by a  $P \times K$  matrix

$$\mathbf{G}_{lq} = [\mathbf{g}_{lq}(0)\mathbf{g}_{lq}(1)\cdots\mathbf{g}_{lq}(K-1)] \quad (6)$$

where

$$\mathbf{g}_{lq}(k) = [\mathbf{g}_{lq}(k)\mathbf{g}_{lq}(k+1/P)\cdots\mathbf{g}_{lq}(k+(P-1)/P)]^T \quad (7)$$

is a  $P \times 1$  vector containing the oversamples associated with the  $k$ -th component of the temporal signature vector of the  $l$ -th path of the  $q$ -th user. In (7),  $\mathbf{g}_{lq}(k)$  is the short notation for  $\mathbf{g}(k - lq)$ . In this case the scalar  $x_m(n)$  in (1) turns into a  $P \times 1$  vector  $\mathbf{x}_m(n)$  that can be written after noise addition as

$$\begin{aligned} \mathbf{y}_m(n) &= \sum_{q=1}^Q \sum_{l=1}^L \sum_{k=0}^{K-1} lq a_m(lq) \mathbf{g}_{lq}(k) s_q(n-k) + \mathbf{v}_m(n) \\ &= \sum_{q=1}^Q \sum_{l=1}^L lq a_m(lq) \mathbf{G}_{lq} \mathbf{s}_q(n) + \mathbf{v}_m(n). \end{aligned} \quad (8)$$

Collecting  $N$  consecutive samples of  $\mathbf{y}_m(n)$ ,  $n = 1, \dots, N$ , the received signal can be expressed as a  $P \times N$  matrix  $\mathbf{Y}_{m..}$  that admits the following factorization

$$\mathbf{Y}_{m..} = \mathbf{G} \mathbf{D}_m [(\mathbf{H} \quad \mathbf{I})] (\mathbf{S}) + \mathbf{V}_{m..}, \quad m = 1, \dots, M \quad (9)$$

where  $\mathbf{H} = \mathbf{A}\mathbf{B}$  is a  $M \times LQ$  spatial signature matrix,  $\mathbf{G} = [\mathbf{G}_{11} \cdots \mathbf{G}_{1q} \cdots \mathbf{G}_{LQ}]$  is a  $P \times KLQ$  temporal signature matrix and  $\mathbf{I}$  and  $\mathbf{S}$  are constraint matrices composed of 1's and 0's entries and defined as

$$\begin{aligned} \mathbf{I} &= \mathbf{I}_{LQ} \otimes \mathbf{1}_K^T \quad (LQ \times KLQ) \\ \mathbf{S} &= \mathbf{I}_Q \otimes \mathbf{1}_L \otimes \mathbf{I}_K \quad (KLQ \times KQ) \end{aligned} \quad (10)$$

$\mathbf{1}_K$  being a ‘‘all ones’’ column vector of dimension  $K \times 1$ . With respect to the PARAFAC decomposition in Section 3, Equation (9) can be interpreted as the  $m$ -th matrix slice of a  $(M, P, N)$ -dimensional tensor  $\mathcal{Y}$ , obtained by slicing the tensor along its first dimension. Comparing (4) and (9), the received signal follows a third-order PARAFAC model where the three matrix components are  $\mathbf{H}$ ,  $\mathbf{G}$  and  $(\mathbf{S})^T$ , respectively. Therefore, the received signal can be viewed as a PARAFAC tensor where two of three factors (say, the first and the third ones) have a special (constrained) structure, the constraints being defined by matrices  $\mathbf{H}$  and  $\mathbf{S}$ . Two other writings of (9) are possible according to (4). The full tensor information can be defined according to (5).

#### 4.2 Receiver Algorithm

The proposed blind receiver algorithm is divided in two processing stages. In the first stage, co-channel user signals are separated in the 3-D (space  $\times$  time  $\times$  oversampling) domain using an alternating least squares (ALS) algorithm that is similar in spirit to the one proposed in [7]. In the second stage, the symbol sequences are independently recovered in the time-domain using a single-user equalization algorithm based on subspace decomposition [9]. Our receiver can be thought of as a blind multiuser equalizer where user separation and equalization are decoupled.

##### 4.2.1 User separation stage:

For the  $(M, P, N)$  received signal tensor  $\mathcal{Y}$ , the user separation stage is represented by the trilinear alternating least squares (ALS) algorithm that consists in estimating three matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$ . In the presence of additive Gaussian noise, these matrices optimize a maximum likelihood criterion formulated as a set of three independent nonlinear least squares minimization problems:

$$\begin{aligned} \hat{\mathbf{C}} &= \underset{\mathbf{C}}{\operatorname{argmin}} \|\mathbf{X}_1 - (\mathbf{A} \diamond \mathbf{B}) \mathbf{C}\|^2 \\ \hat{\mathbf{A}} &= \underset{\mathbf{A}}{\operatorname{argmin}} \|\mathbf{X}_2 - (\mathbf{B} \diamond (\mathbf{C})^T) (\mathbf{A})^T\|^2 \\ \hat{\mathbf{B}} &= \underset{\mathbf{B}}{\operatorname{argmin}} \|\mathbf{X}_3 - ((\mathbf{C})^T \diamond \mathbf{A}) \mathbf{B}^T\|^2 \end{aligned} \quad (11)$$

where  $\mathbf{X}_1$  ( $PM \times N$ ),  $\mathbf{X}_2$  ( $NP \times M$ ) and  $\mathbf{X}_3$  ( $MN \times P$ ) are matrices formed from the slices of the received signal tensor along the first, second and third dimensions, respectively. After removal of scaling and permutation ambiguities, these matrices will be related to the matrices  $\hat{\mathbf{H}}$ ,  $\hat{\mathbf{G}}$  and  $\hat{\mathbf{S}}$  in the following way:

$$\hat{\mathbf{H}} = \hat{\mathbf{A}}, \quad \hat{\mathbf{G}} = \hat{\mathbf{B}}\mathbf{T}, \quad \hat{\mathbf{S}} = \mathbf{T}^{-1}\hat{\mathbf{C}}, \quad (12)$$

where  $\mathbf{T}$  is a  $KQ \times KQ$  square ambiguity matrix that will be found in the second stage of the receiver. The  $i$ -th iteration of the ALS algorithm consists of three steps: 1) update  $\hat{\mathbf{C}}_i$  conditioned on  $\hat{\mathbf{A}}_{i-1}$  and  $\hat{\mathbf{B}}_{i-1}$ ; 2) update  $\hat{\mathbf{A}}_i$  conditioned on  $\hat{\mathbf{B}}_{i-1}$  and  $\hat{\mathbf{C}}_i$ ; 3) update  $\hat{\mathbf{B}}_i$  conditioned on  $\hat{\mathbf{A}}_i$  and  $\hat{\mathbf{C}}_i$ . These three updating steps are repeated until convergence of the algorithm. Several initialization strategies exist. Here, we initialize  $\hat{\mathbf{A}}_0 = \mathbf{A} + \mathbf{E}_a$  and  $\hat{\mathbf{B}}_0 = \mathbf{B} + \mathbf{E}_b$ , where  $\mathbf{E}_a$  and  $\mathbf{E}_b$  are matrices whose entries are randomly generated from a normal distribution with  $\sigma = 0.01$ . In practice, a good initial guess for  $\hat{\mathbf{A}}_0$  and  $\hat{\mathbf{B}}_0$  can be obtained, for example, from prior information (or imprecise knowledge) about the spatial geometry of the receiver antenna array as well as from knowledge about the temporal structure of the pulse-shaping filters. If no prior knowledge is available, initialization can be done from the matrix slices of the received signal via generalized eigenvalue decomposition methods. Other more sophisticated initialization strategies exist but they are beyond the scope of this work.

##### 4.2.2 Equalization stage:

At the end of the first stage of the receiver, we are left with matrices  $\hat{\mathbf{A}} = \hat{\mathbf{H}}$ ,  $\hat{\mathbf{B}} = \hat{\mathbf{G}}\mathbf{T}^{-1}$  and  $\hat{\mathbf{C}} = \mathbf{T}\hat{\mathbf{S}}$ . In order to estimate the transmitted sequences we must determine the ambiguity matrix  $\mathbf{T}$ . According to (12), the transmitted sequences can be recovered by solving  $\hat{\mathbf{C}} = \mathbf{T}\hat{\mathbf{S}}$ , i.e.,

$$\begin{bmatrix} \hat{\mathbf{C}}_1 \\ \vdots \\ \hat{\mathbf{C}}_Q \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & & \\ & \ddots & \\ & & \mathbf{T}_Q \end{bmatrix} \begin{bmatrix} \hat{\mathbf{S}}_1 \\ \vdots \\ \hat{\mathbf{S}}_Q \end{bmatrix}. \quad (13)$$

The ambiguity matrix  $\mathbf{T}$  is block-diagonal, i.e.  $\mathbf{T} = \mathit{BlockDiag}(\mathbf{T}_1 \cdots \mathbf{T}_Q)$ , which means that users' symbol sequences can be independently recovered by solving a set of smaller system of equations  $\hat{\mathbf{C}}_q = \mathbf{T}_q \hat{\mathbf{S}}_q$ ,  $q = 1, \dots, Q$ , each

one of which being a single-input multiple-output (SIMO) blind equalization problem. In other words, the PARAFAC approach decouples a multiuser equalization problem into  $Q$  equivalent single-user equalization problems. The  $K \times K$  square (non-singular) ambiguity matrix  $\mathbf{T}_q$ ,  $q = 1, \dots, Q$ , can be interpreted as an equivalent SIMO channel with  $K$  impulse responses of length  $K$  each. Several strategies exist that can be used to blindly estimate  $\mathbf{T}_q$  and  $\mathbf{S}_q$ ,  $q = 1, \dots, Q$ . Here we use the subspace algorithm proposed by Moulines et al. [9], in which  $\mathbf{T}_q$  is found by minimizing the following quadratic cost function

$$\mathbf{t}_q = \underset{\mathbf{t}_q}{\operatorname{argmin}} \mathbf{t}_q^H \mathcal{F}(\mathbf{U}_q) \mathcal{F}(\mathbf{U}_q)^H \mathbf{t}_q \quad (14)$$

under the constraint  $\|\mathbf{t}_q\| = 1$ , for each  $q = 1, \dots, Q$ , where  $\mathbf{t}_q = \operatorname{vec}(\mathbf{T}_q)$  and  $\mathcal{F}(\mathbf{U}_q)$  is a block-Toeplitz matrix formed from a basis of the noise subspace  $\mathbf{U}_q$  (associated with the smallest left singular vectors) of a convolution matrix formed from  $\mathbf{C}_q$ . For reasons of space, we report the interested reader to [9] for further details. After estimation of  $\mathbf{T}_q$ , we calculate  $\hat{\mathbf{S}}_q = \mathbf{T}_q^{-1} \hat{\mathbf{C}}_q$  for each  $q = 1, \dots, Q$ .

## 5. SIMULATION RESULTS

The average bit-error-rate (BER) performance of the proposed blind PARAFAC multiuser equalization receiver has been evaluated through computer simulations, considering  $Q = 2$  co-channel users. The signal transmitted by each user arrives at the receiver via  $L = 2$  independent Rayleigh-faded multipaths. The multipaths of the first user are parameterized by  $[\alpha_{11}, \alpha_{21}] = [0^\circ, 30^\circ]$ ,  $[\alpha_{11}, \alpha_{21}] = [0, T]$  and  $[\alpha_{11}, \alpha_{21}] = [1, 1]$ , while those of the second user are parameterized by  $[\alpha_{12}, \alpha_{22}] = [-20^\circ, 50^\circ]$ ,  $[\alpha_{12}, \alpha_{22}] = [0, T]$  and  $[\alpha_{12}, \alpha_{22}] = [1, 1]$ . The length of the channel impulse response is  $K = 2$  symbols. For all the simulations and receivers, the number of receiver antennas is  $M = 2$ , the oversampling factor is  $P = 2$  and the number of received binary-phase shift keying (BPSK) symbols processed is  $N = 50$ . BER results are plotted as a function of the signal-to-noise ratio (SNR) per receiver antenna and are drawn from 1000 independent Monte Carlo runs.

In Figure 1, we compare the results of the blind PARAFAC receiver with those of previously proposed blind subspace-based receivers. For the multiuser case, the blind PARAFAC receiver is compared with the blind subspace algorithm proposed by Van der Veen et al. [3], which combines a subspace method with the exploitation of the Finite-Alphabet (FA) property of the transmitted symbols for multiuser space-time equalization. This receiver is called "Subspace+FA" in the figure. We have also plotted the classical subspace receiver by Moulines et al. [9] as a reference for comparison. For the classical subspace receiver, we have assumed  $Q = 1$ , since it is a single-user receiver. This receiver is called "Classical Subspace" in the figure. It can be seen that the proposed PARAFAC receiver outperforms the Subspace+FA receiver in the multiuser frequency-selective scenario with  $L = 2$  and  $K = 2$ . The PARAFAC receiver is also close to the single-user subspace receiver, with a performance gap of only 3dB for a target BER of  $10^{-4}$ . For the a single-path flat fading scenario ( $K=L=1$ ) scenario (only the first multipath of each user is present), the performance improvement of the proposed PARAFAC receiver over the Subspace+FA one is also verified (5dB for  $10^{-2}$  target BER).

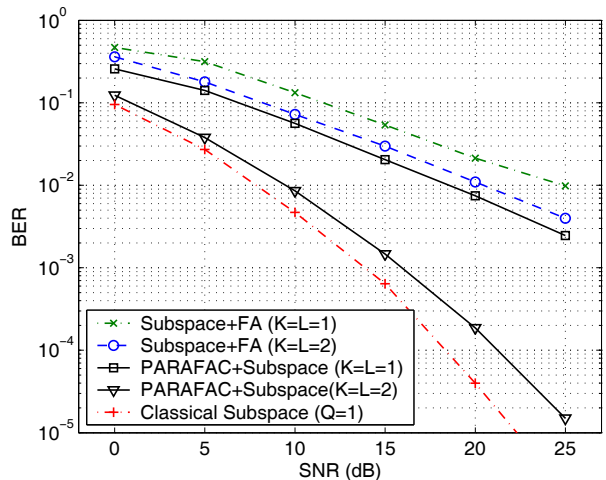


Figure 1: Performance of the PARAFAC receiver, compared with two blind subspace receivers.  $M = 2$ ,  $P = 2$  and  $N = 50$ .

## 6. CONCLUSIONS

In this work we have proposed a blind PARAFAC receiver for multiuser equalization in antenna array-based wireless communication systems with temporal oversampling. The proposed receiver is based on a PARAFAC modelling approach for the received signal. The developed PARAFAC model has assumed a parametric multipath channel model with frequency-selective fading. Based on this model, we have proposed a blind PARAFAC receiver that performs signal separation and equalization in two processing stages. In the first one, user signals are separated in the 3-D (space  $\times$  time  $\times$  oversampling)-domain using the ALS algorithm. In the second stage, a subspace method is used to independently equalize each user sequence. Simulation results were provided to compare the BER performance of the proposed receiver with that of classical ones.

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