COMPLEX-VALUED FIR SEISMIC MIGRATION FILTER DESIGN USING PURE AND RELAXED PROJECTION ALGORITHMS

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ABSTRACT

The iterative Pure and Relaxed Projectors are special cases of Vector Space Projection Methods (VSPM). Complex-valued FIR digital filter design using Pure and Relaxed Projectors is proposed for designing seismic migration digital filters. Designing FIR digital filters using VSPM, in general, is able to produce feasible solutions satisfying all the desired filter constraints by the use of only two Fast Fourier Transform (FFT) computations. Furthermore, the design algorithm can be directly extended from one-dimensional (1-D) to multi-dimensional (m-D) filters, an advantage not present in many other filter design techniques. In addition, by using the relaxed projectors, one can achieve faster convergence when compared to the pure projectors. The simulation results show that the relaxed projectors for designing such filters will save up to 86.47% of the number of iterations when compared with the pure projectors.

1. INTRODUCTION

Among many geophysical surveying techniques, seismic reflection surveying is the most widely used and well-known geophysical technique. Seismic reflection acquired data can be produced to reveal details of geological structures on scales from the top tens of meters of drift to the whole lithosphere [1]. Part of its success lies in the fact that the raw seismic data is processed to produce a seismic section which is an image of the subsurface structure. However, the layers in seismic sections are incorrectly positioned and the resolution of such sections are also affected due to factors related to the geological structure of the earth itself [1, 2]. *Seismic Migration Filters*, which are basically non-causal complex-valued filters with non-linear phases [2], can correct such undesirable effects.

An iterative FIR filter design algorithm based on one forward and one backward Fast Fourier transforms (FFT) to design zero-phase FIR filters was introduced in [3]. The algorithm alternatively satisfies the frequency domain constraints on the magnitude response bounds as well as time domain constraints on the impulse response support [3, 4]. The main advantages of this method are based on its implementation simplicity and versatility. This algorithm is essentially a special case of Vector Space Projection Methods (VSPM) which is known as *Projection onto Convex Sets (POCS)* or *Pure Projectors*. However, the algorithm was derived heuristically without explicitly defining the constraint sets properly and deriving their associated projections. In addition, the heuristic nature of such approach does not obviously lend itself to the design of filters with other constraints [5].

In [5], the proper mathematical way to derive a design algorithm for real-valued linear-phase FIR filters using POCS was shown. Although the VSPM theory, in general, does not yield an optimal solution, it does, however, lead to a feasible solution satisfying all predefined constraint sets. For the POCS theory, this is based on constraint sets which satisfy certain conditions like being closed and convex in a Hilbert space [5]. In [5] the Hilbert space was the M-dimensional Euclidean space. Moreover, unlike many FIR filter design techniques, the VSPM for designing FIR filters can easily be directly extended to the design of m-D filters [5, 4]. Furthermore, for the case of designing FIR digital filters, the VSPM will require only two FFT computations per iteration [3]. Also, one can speed up the convergence of such algorithms by using what is known as *relaxed projectors* [6]. This is important when designing seismic migration filters since migration is a recursive process and will require a filter for each given frequency/velocity parameter [1, 2]. Moreover, new digital seismic acquisition systems can directly process seismic data before storing it by incorporating digital filters in the system themselves. Hence, one is interested in designing complex-valued FIR seismic migration filters which will be based on the 1-D and 2-D Migration filter desired specifications using VSPMs or more precisely, by using the pure and relaxed projection algorithms.

Seismic migration for 2-D seismic wavefields will be introduced in section 2. Section 3, will state the fundamental theorem of POCS for both the relaxed and the pure projectors. Section 4 deals with the design of such filters using VSPMs by setting up the required constraint sets. The design algorithm is presented in section 5 for both the pure and the relaxed projection methods. Finally, section 6 presents some results which can be used for 2-D and 3-D seismic migration processes.

2. SEISMIC MIGRATION

Let $u(x,t,z)$ be an acoustic seismic wave which is propagating upwards though the earth with a uniform acoustic velocity c[1]. This wave is governed by the following 2-D hyperbolic wave equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}. \tag{1}$$

Eq. (1) occurs in the field of seismic data processing where the recorded data is given at a reference surface $z = 0$, i.e., $u(x,t,0)$, and we wish to evaluate the downward continued section at a depth z_o, i.e., $u(x,t,z_o)$. The solution of Eq. (1) can be found using the 2-D Fourier transform. In practice, the acoustic wave $u(x,t,z)$ is presented in sampled form as $u(n_1\Delta x, n_2\Delta t, n_3\Delta z)$ where n_1, n_2, and n_3 belong to the set of integers, Δt is the temporal sampling interval, Δx is the horizontal spatial sampling interval, and, finally, Δz is the vertical spatial sampling interval. The 2-D migration operation frequency response is given by

$$H_d(e^{jk_x}, e^{j\omega}) = \exp[j\frac{\Delta z}{\Delta x}\sqrt{\frac{\Delta x^2}{\Delta t^2}\frac{\omega^2}{c^2} - k_x^2}] \tag{2}$$

which is the ideal frequency response of an all-pass filter with non-linear phase. The process is carried out for each fixed ratio ω_o/c_o

[1]By fixing the velocity, one is assuming that the wave is propagating in a homogeneous medium [1].

at a time. Hence, Eq. (2) becomes the following 1-D filter:

$$H_d(e^{jk_x}, e^{j\omega_o}) = H_d(e^{jk_x}) = \exp[j\frac{\Delta z}{\Delta x}\sqrt{\frac{\Delta x^2}{\Delta t^2}\frac{\omega_o^2}{c_o^2} - k_x^2}] \quad (3)$$

with a cut-off at the wavenumber $k_x = \frac{\Delta x}{\Delta t}\frac{\omega_o}{c_o}$. An important remark that follows from Eq. (3) is that $H_d(e^{jk_x})$ is a complex-valued even function. Also, it was shown in [7] that Eq. (3) cannot be obtained by a causal filter since its impulse response is defined for negative values of the variable t. Therefore, Eq. (3) could be approximated by a non-causal even symmetric N-length FIR digital filter [2] given by:

$$H_d(e^{jk_x}) = h[0] + 2\sum_{n=1}^{\frac{N+1}{2}-1} h[n]\cos(nk_x) \quad (4)$$

where $h[n] \in \mathbb{C}$, i.e., the FIR filter coefficients are complex-valued. Now, by setting $k_c = \frac{\Delta x}{\Delta t}\frac{\omega_o}{c_o}$ the approximation needs to be most accurate for $|k_x| < |k_c|$. This corresponds to the wavenumbers k_x for which the waves are propagating. Finally, let $b = \Delta z/\Delta x$, and then the 1-D migration filter simply becomes:

$$H_d(e^{jk_x}) = \exp[jb\sqrt{k_c^2 - k_x^2}]. \quad (5)$$

3. VECTOR SPACE PROJECTION METHODS (VSPM)

Assume that $C_1, C_2, \cdots, C_m$ denote m closed convex sets in a Hilbert space $\mathbf{H}$ and let C_o denote their intersection set given by:

$$C_o = \bigcap_{i=1}^{m} C_i. \quad (6)$$

For each $i = 1, 2, \cdots, m$, let P_{C_i} denote the projection operator onto the set C_i and let I be an identity operator, then the corresponding relaxed projector T_{C_i} is given by [6]:

$$T_{C_i} = I + \lambda_{C_i}(P_{C_i} - I) \quad (7)$$

where λ_i is defined to be the relaxation parameter and has values in the range $(0, 2)$. Also, let T refer to the concatenation of all these relaxed projectors, that is,

$$T = T_{C_m} T_{C_{m-1}} \cdots T_{C_1}. \quad (8)$$

Then, the following is the Fundamental Theorem of POCS [6]:

Theorem 3.1 *Assume that C_o is non-empty. Then for every $\mathbf{h} \in \mathbf{H}$ and for every $\lambda_i \in (0, 2)$, $i = 1, 2, \cdots, m$, the sequence $\{T_{C_n}\mathbf{h}\}$ converges weakly to a point of C_o.*

In other words, theorem 3.1 states that the iterates $\{\mathbf{h}_k\}$ generated by

$$\mathbf{h}_{k+1} = T_{C_m} T_{C_{m-1}} \cdots T_{C_1}\mathbf{h}_k \quad (9)$$

with an arbitrary starting point $\mathbf{h}_0$ will converge weakly to a point of C_o and since our Hilbert space is of finite dimension, the algorithm will strongly converge to a point of C_o. This algorithm is generally referred to as *Relaxed Projection*. In particular, when the projector P_{C_i} is used instead of T_{C_i} for each set in Eq. (9), the algorithm reduces to:

$$\mathbf{h}_{k+1} = P_{C_m} P_{C_{m-1}} \cdots P_{C_1}\mathbf{h}_k \quad (10)$$

This will be referred to as the *Pure Projection* algorithm [6].

4. COMPLEX-VALUED FIR SEISMIC MIGRATION FILTER DESIGN USING VSPM

As shown in section 2, seismic migration filter coefficients are complex-valued. This implies that our Hilbert space must be the set of complex-valued M-dimensional vectors, i.e., $\mathbb{C}^M$ where $M \gg N$. The designed seismic migration filter must satisfy both the space and the wavenumber domain requirements in which these requirements are put in proper sets that are closed and convex. These requirements are summarized as follows:

1. N-length complex-valued FIR filter, and having even symmetry, meaning that N must be odd [2].
2. In the passband, the magnitude spectrum of its discrete space Fourier transform (DSFT) must be upper and lower bounded by $1+\delta_p$ and $1-\delta_p$, respectively. In addition, the stopband magnitude spectrum must be bounded by δ_s. Finally, its phase spectrum must have even symmetry, as in Eq. (5).

In order to do so, what follow are the constraint sets which will be closed and convex, and their intersection will not be empty, i.e., we should be able to design such filters with the predefined characteristics. These constraint sets are:

1. $C_1 = \{\mathbf{h} \in \mathbb{C}^M : h[n] = h[-n] \text{ for } n \in S \text{ and } h[n] = 0 \text{ for } n \in S^c\}$ where $S = \{-\frac{N-1}{2}, -\frac{N-1}{2}+1, \cdots, 0, \cdots, \frac{N-1}{2}-1, \frac{N-1}{2}\}$ and N is odd, and S^c is its complement. In words, C_1 is the set of all complex-valued sequences (vectors) of length M with at most N odd non-zero members (filter coefficients) that are non-causal and having even symmetry.

2. $C_2 = \{\mathbf{h} \in \mathbb{C}^M \text{ with } h[n] \leftrightarrow H(e^{jk_x}) : \angle H(e^{jk_x}) = \phi(k_x) = b\sqrt{k_{c_p}^2 - k_x^2}\}$ where k_{c_p} is the cut-off wavenumber. That is, C_2 is basically the set of all sequences which are complex-valued and whose DSFT argument (phase response) is constrained to be equal to $b\sqrt{k_{c_p}^2 - k_x^2}$.

3. $C_3 = \{\mathbf{h} \in \mathbb{C}^M \text{ with } h[n] \leftrightarrow H(e^{jk_x}) : |H(e^{jk_x})| \geq 1-\delta_p \text{ for } k_x \in k_{x_p}\}$ where k_{x_p} is the passband interval which is equal to $[-k_{c_p}, k_{c_p}]$ and δ_p is the maximum passband allowable tolerance. C_3 is the set of complex-valued sequences whose DSFT magnitude spectrum is lower bounded by $1-\delta_p$ in the passband.

4. $C_4 = \{\mathbf{h} \in \mathbb{C}^M \text{ with } h[n] \leftrightarrow H(e^{jk_x}) : |H(e^{jk_x})| \leq 1+\delta_p \text{ for } k_x \in k_{x_p}\}$. In words, C_4 is the set of complex-valued sequences whose DSFT magnitude should not exceed the limit $1+\delta_p$ in the passband.

5. $C_5 = \{\mathbf{h} \in \mathbb{C}^M \text{ with } h[n] \leftrightarrow H(e^{jk_x}) : |H(e^{jk_x})| \leq \delta_s \text{ for } k_x \in k_{x_s}\}$ where $k_{x_s} = [-\pi, -k_{c_s}) \cap (k_{c_s}, \pi]$. k_{c_s} is the stopband cut-off wavenumber and δ_s is the maximum allowable stopband tolerance. So, C_5 is the set of all sequences which are complex-valued and whose DSFT magnitude is limited to a maximum of δ_s in the stopband k_{x_s}.

One can show that all the above constraint sets are closed and convex sets. Also, it can be shown that the associated projection of an arbitrary vector $\mathbf{h} \in \mathbb{C}^M$, where $\mathbf{h} \notin C_i$ for $i = 1, \cdots, 5$ is given by:

$$P_{C_1}\mathbf{h} = \begin{cases} h[n], \text{ for } |n| \leq (N-1)/2 \\ 0, \text{ otherwise} \end{cases} \quad (11)$$

$$P_{C_2}\mathbf{h} \leftrightarrow \begin{cases} |H(e^{jk_x})|\cos(\theta_h - \phi(k_x))\exp(j\phi(k_x)), \text{ if } A_{C_2} \\ -|H(e^{jk_x})|\cos(\theta_h - \phi(k_x))\exp(j\phi(k_x)), \text{ if } B_{C_2} \end{cases} \quad (12)$$

where $\theta_h = \angle H(e^{jk_x})$; condition A_{C_2} is $\cos(\theta_h - \phi(k_x)) \geq 0$; and condition B_{C_2} is $\cos(\theta_h - \phi(k_x)) < 0$.

$$P_{C_3}\mathbf{h} \leftrightarrow \begin{cases} H(e^{jk_x}), \text{ if } A_{C_3} \\ (1-\delta_p)\exp(j\angle H(e^{jk_x})), \text{ if } B_{C_3} \\ H(e^{jk_x}), \text{ otherwise} \end{cases} \quad (13)$$

where condition A_{C_3} is $|H(e^{jk_x})| > (1-\delta_p)$, for $k_x \in k_{x_p}$; and condition B_{C_3} is $|H(e^{jk_x})| \leq (1-\delta_p)$, for $k_x \in k_{x_p}$.

$$P_{C_4}\mathbf{h} \leftrightarrow \begin{cases} H(e^{jk_x}), \text{ if } A_{C_4} \\ -(1+\delta_p)\exp(j\angle H(e^{jk_x})), \text{ if } B_{C_4} \\ H(e^{jk_x}), \text{ otherwise} \end{cases} \tag{14}$$

where condition A_{C_4} is $|H(e^{jk_x})| < (1+\delta_p)$, for $k_x \in k_{x_p}$; and condition B_{C_4} is $|H(e^{jk_x})| \geq (1+\delta_p)$, for $k_x \in k_{x_p}$.

$$P_{C_5}\mathbf{h} \leftrightarrow \begin{cases} H(e^{jk_x}), \text{ if } A_{C_5} \\ -\delta_s \exp(j\angle H(e^{jk_x})), \text{ if } B_{C_5} \\ H(e^{jk_x}), \text{ otherwise} \end{cases} \tag{15}$$

where condition A_{C_5} is $|H(e^{jk_x})| < \delta_s$, for $k_x \in k_{x_s}$; and condition B_{C_5} is $|H(e^{jk_x})| \geq \delta_s$, for $k_x \in k_{x_s}$.

5. COMPLEX-VALUED FIR MIGRATION FILTER DESIGN ALGORITHM USING VSPM

5.1 The Pure Projection Design Algorithm

So from Eq. (10), the projection design algorithm for complex-valued FIR seismic migration filters is:

$$\mathbf{h}_{k+1} = P_{C_1}P_{C_2}P_{C_3}P_{C_4}P_{C_5}\mathbf{h}_k \tag{16}$$

where P_{C_1}, P_{C_2}, P_{C_3}, P_{C_4}, and P_{C_5} are given in Eqs. (11), (12), (13), (14), and (15), respectively.

5.2 The Relaxed Projection Design Algorithm

As mentioned earlier in section 1, one can speed up the convergence of the pure projection algorithm by using the relaxed projectors given in Eqs. (7) and (9). In addition, recall that when the relaxation parameter λ_{C_i} for the constraint set C_i is equal to 1, the relaxed projector simply becomes the pure projector. Also, λ_{C_i} must lie in the interval $(0,2)$. It is reported that the relaxation parameters for more than two constraint sets can only be determined heuristically [6]. However, one can have an idea of where their values might lie based on the derived pure projectors. For all the previously derived projectors, these intervals where determined and after several experiments, it turns out that the most important projectors which can significantly reduce the number of iterations for designing these filters are the constraint sets C_2 and C_3. So by fixing $\lambda_1 = \lambda_4 = \lambda_5 = 1$, and varying the other parameters over their allowed range, one expects to achieve faster convergence with the relaxed projection algorithm compared to the pure projection method. So, the following is the relaxed algorithm version of Eq. (16):

$$\mathbf{h}_{k+1} = P_{C_1}T_{C_2}T_{C_3}P_{C_4}P_{C_5}\mathbf{h}_k \tag{17}$$

where T_{C_2} and T_{C_3} are the relaxed projectors associated with C_2 and C_3, respectively. The following subsections deal with the range of λ_{C_2}'s and λ_{C_3}'s for which the algorithm in Eq. (17) will converge faster than the algorithm given in Eq. (16).

5.2.1 Relaxed Projector for C_2

Recall that the projection onto C_2 was given by Eq. (12), hence from Eq. (7) we can write the relaxed version of this projector as:

$$T_{C_2}\mathbf{h} \leftrightarrow \begin{cases} (1-\lambda_{C_2})H(e^{jk_x}) + \lambda_{C_2}P_{C_2}H, \text{ if } A_{C_2} \\ (1-\lambda_{C_2})H(e^{jk_x}) - \lambda_{C_2}P_{C_2}H, \text{ if } B_{C_2} \end{cases} \tag{18}$$

where $P_{C_2}H = |H(e^{jk_x})|\cos(\theta_h - \phi(k_x))\exp(j\phi(k_x))$, the condition A_{C_2} represents $\cos(\theta_h - \phi(k_x)) \geq 0$, and the condition B_{C_2} represents $\cos(\theta_h - \phi(k_x)) < 0$. We can show that for $\cos(\theta_h - \phi(k_x)) \geq 0$ then $\lambda_{C_2} \in (0,1)$ if $\cos(\phi(k_x)) \geq 0$ or $\sin(\phi(k_x)) \geq 0$. Otherwise, $\lambda_{C_2} = 1$. Alternatively, for $\cos(\theta_h - \phi(k_x)) < 0$, one can show that if $\cos(\phi(k_x)) \leq 0$ or $\sin(\phi(k_x)) \leq 0$, then $\lambda_{C_2} \in (0,1)$. Otherwise, $\lambda_{C_2} = 1$.

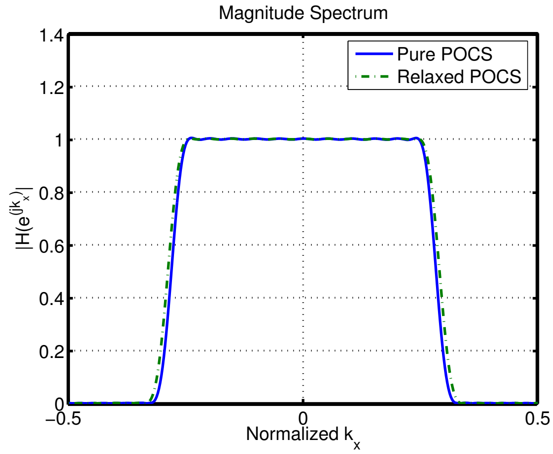

Figure 1: Magnitude response of the designed 1-D seismic migration filter using pure POCS (solid line) and relaxed POCS (dashed line) with $N = 39$, $k_{c_p} = 0.25$, $k_{c_s} = 0.3347$, $\delta_p = \delta_s = 10^{-3}$, and $\varepsilon = 10^{-12}$.

5.2.2 Relaxed Projector for C_3

Recall that the projection onto C_3 (i.e., P_{C_3}) was given by Eq. (13) and again by using Eq. (7), one can relax Eq. (13) to get

$$T_{C_3}\mathbf{h} \leftrightarrow \begin{cases} H(e^{jk_x}), \text{ if } A_{C_3} \\ (1-\lambda_{C_3})H(e^{jk_x}) + \lambda_{C_3}P_{C_3}H, \text{ if } B_{C_3} \\ H(e^{jk_x}), \text{ otherwise} \end{cases} \tag{19}$$

where $P_{C_3}H = (1-\delta_p)\exp(j\angle H(e^{jk_x}))$, the condition A_{C_3} represents $|H(e^{jk_x})| > (1-\delta_p)$ and $k_x \in k_{x_p}$, and the condition B_{C_3} represents $|H(e^{jk_x})| \leq (1-\delta_p)$ and $k_x \in k_{x_p}$. Again, we can show that by selecting $\lambda_{C_3} \in (1,2)$, we will achieve a faster converging algorithm.

In general, the pure and the relaxed projection seismic migration design algorithms can explicitly be stated as follows:
Start with an arbitrary complex-valued vector of dimension M, called $\mathbf{h}_0$. Then for the kth iteration:

1. Take the FFT of $\mathbf{h}_k$.
2. Impose the bounds in the Fourier domain given by Eqs. (12), (13), (14), (15) for the pure projector or Eqs. (18), (19), (14), (15) for the relaxed projector.
3. Take the inverse FFT of the output from the previous step.
4. Finally, impose the space-domain constraint given in Eq. (11) which will yield the output vector $\mathbf{h}_{k+1}$.

If the mean-square error between $\mathbf{h}_{k+1}$ and $\mathbf{h}_k$is less than or equal to a predefined threshold ε, then stop the algorithm. Otherwise, repeat steps 1-4.

6. SIMULATION RESULTS

A 39-tap 1-D complex-valued FIR seismic migration digital filter is designed using the pure projector (pure POCS) algorithm and the relaxed projector (relaxed POCS) where both algorithms were described in section 5. The filter parameters are: $k_{c_p} = 0.25$, $k_{c_s} = 0.3347$, and $\delta_p = \delta_s = 10^{-3}$. Fig. 1 shows the magnitude spectrum for the designed filters using both the pure and relaxed projection algorithms, respectively. Clearly, in both cases, an even symmetrical magnitude spectrum with respect to $k_x = 0$ axis was obtained meeting the filter magnitude spectrum constrains, i.e., C_3, C_4, and C_5. And Fig. 2 shows the phase spectrum in the passband for the designed filters using both the pure and relaxed projection algorithms where the phase in the passband meets the constraint C_2 requirement, i.e., having even symmetry with respect to $k_x = 0$ axis

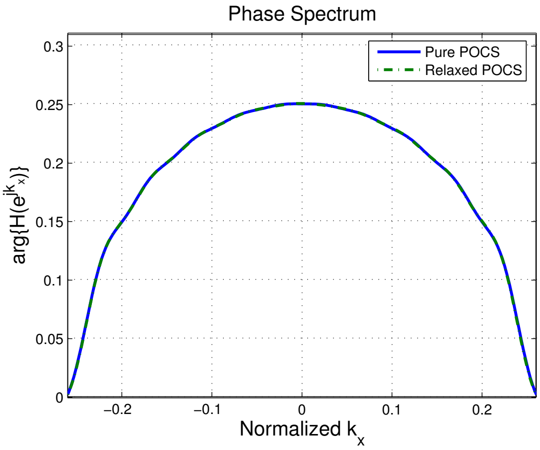

Figure 2: Phase response of the designed 1-D seismic migration filter using pure POCS (solid line) and relaxed POCS (dashed line) with $N = 39$, $k_{c_p} = 0.25$, $k_{c_s} = 0.3347$, $\delta_p = \delta_s = 10^{-3}$, and $\varepsilon = 10^{-12}$.

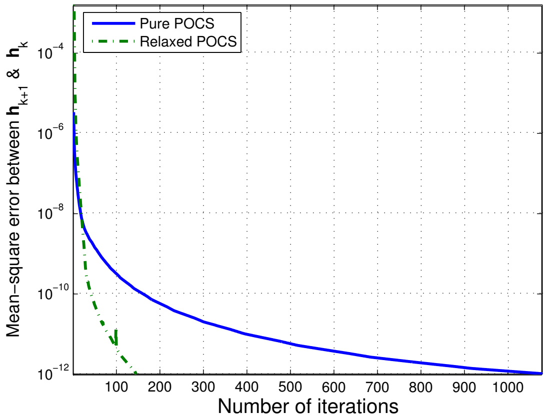

Figure 3: The mean-square error between $\mathbf{h}_{k+1}$ and $\mathbf{h}_k$, versus the number of iterations, for the designed 1-D seismic migration filter using pure POCS and relaxed POCS with $N = 39$, $k_{c_p} = 0.25$, $k_{c_s} = 0.3347$, $\delta_p = \delta_s = 10^{-3}$, and $\varepsilon = 10^{-12}$.

and approximately equal to the desired phase spectrum. For the impulse response, since it has even symmetry, it is enough to show only half the coefficients as has been done in Table. 1. Here the real and imaginary parts of the complex-valued impulse response are shown for both the pure and relaxed projectors. The coefficients (solutions) are not identical but they both satisfy the wavenumber response constraint sets, i.e., both yield two different solutions for the same solution set C_o. Finally, for the same filter parameters, as shown in Fig. 3, the pure projection design algorithm took 1078 iterations to converge while the relaxed projection design algorithm took only 146 iterations to converge, both with respect to a mean-square error threshold $\varepsilon = 10^{-12}$. In this case, this means that for the same filter requirements and parameters, and for the same stopping threshold, the relaxed projection algorithm is saving 86.47% of the number of iterations when compared to the pure projection algorithm.

7. CONCLUSION

The pure and relaxed projectors have been derived and used to design complex-valued FIR seismic migration filters where two FFT computations per iteration are required for the design. The designed filters satisfy all seismic migration filter requirements, although they need not to be optimal. Based on the simulation results, the relaxed projection algorithm has significantly reduced the number of iterations by 86.47%, when compared to the pure projection algorithm for the same filter parameters and for the same stopping threshold.

Table 1: FIR filter coefficients for both the pure POCS & the relaxed POCS with $N = 39$, $k_{c_p} = 0.25$, $k_{c_s} = 0.3347$, $\delta_p = \delta_s = 10^{-3}$, and $\varepsilon = 10^{-12}$.

Index	*PurePOCS*		*RelaxedPOCS*	
n	$\Re h[n]$	$\Im h[n]$	$\Re h[n]$	$\Im h[n]$
0	0.553	0.0972	0.568	0.0971
1	−0.303	−0.0702	−0.299	−0.0701
2	−0.0629	0.0177	−0.0757	0.0177
3	0.0839	0.0116	0.0748	0.0116
4	0.053	−0.0067	0.0604	−0.00674
5	−0.0309	−0.00523	−0.0191	−0.005211
6	−0.0409	0.00376	−0.0425	0.00379
7	0.0068	0.00309	−0.00429	0.00307
8	0.0285	−0.00249	0.0249	−0.00253
9	0.0044	−0.00209	0.0122	−0.00207
10	−0.0172	0.0018	−0.0111	0.00185
11	−0.00812	0.00153	−0.0117	0.00149
12	0.00857	−0.00139	0.00237	−0.00144
13	0.0077	−0.00119	0.00784	−0.00114
14	−0.00302	0.00111	0.00149	0.00116
15	−0.00542	0.000945	−0.00384	0.000893
16	0.000258	−0.000926	−0.00021	−0.000977
17	0.00306	−0.00078	0.00123	−0.000714
18	0.000688	0.0009	0.00151	0.000865
19	−0.00146	0.000615	−0.000307	0.00048

8. ACKNOWLEDGMENT

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