ADAPTIVE LAGUERRE-VOLTERRA FILTERS OPTIMIZATION BASED ON LAGUERRE SPECTRA

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ABSTRACT

In this paper, the optimization of Laguerre-Volterra filters (LVFs) is carried out adaptively. Each kernel is expanded on an independent Laguerre basis. An analytical solution to Laguerre poles optimization is provided using the knowledge of the expansion coefficients, also called Fourier coefficients, associated with an arbitrary Laguerre basis. These coefficients are estimated by means of the Normalized Least Mean Squares (NLMS) algorithm. The proposed method allows optimization of both the Fourier coefficients and the Laguerre poles.

1. INTRODUCTION

Truncated Volterra series models have become very popular in adaptive nonlinear filtering applications [1]. However the main drawback of these filters is their over-parameterization. During the last decade, the issue of Volterra model complexity reduction has been addressed using various approaches. The expansion of Volterra kernels on discrete orthonormal bases of functions (OBF) is one of them [2, 3, 4]. The class of OBF generally used for modelling purposes is that of rational orthonormal bases such as Laguerre basis.

The Laguerre functions have the property to be completely characterized by a single parameter, the Laguerre pole. When expanding a Volterra kernel on a Laguerre basis, the parsimony of the expansion is strongly linked to the choice of the Laguerre pole. Although expansion of Volterra kernels on Laguerre basis was firstly suggested by Wiener in the 50's [5], in the best of our knowledge Campello *et al.*[3] were the first to derive an analytical solution to the Laguerre pole optimization for Volterra models. They generalized the work in [6] and also showed that using independent bases to expand each kernel gives better results than the use of a single basis. However the obtained analytical solution is based on the knowledge of the Volterra kernels. Consequently a step of Volterra kernels estimation is needed before the application of this solution. Note again that this method is applicable if and only if the kernels satisfy the unit delay condition.

In order to circumvent these limitations a new approach is proposed in this paper. It is based on the knowledge of the estimated expansion coefficients, also called Fourier coefficients associated with an arbitrary Laguerre basis. The requirement of a unit delay is relaxed. This approach can be viewed as a generalization of both [7] and [3].

The organization of the paper is as follows. In the next section, the principle of Volterra model expansion on OBF is recalled and the expression of the analytical optimal Laguerre pole is given for each kernel. In section 3 this pole is expressed in terms of the Laguerre spectrum, i.e. the set of Fourier coefficients relative to the corresponding kernel. Then the adaptive identification method is described in section 4 and illustrated by means of simulation results in section 5 before concluding the paper in section 6.

2. BACKGROUND

A discrete-time P-th order Volterra filter is described by the following input-output relation:

$$y(n) = \sum_{p=1}^{P} \sum_{n_1=0} \cdots \sum_{n_p=0} h_p(n_1, \cdots, n_p) \prod_{j=1}^{p} u(n-n_j), \qquad (1)$$

where u, y and h_p are respectively the input, the output and the p-th order Volterra kernel. It has been shown that any causal non-linear system, time invariant with fading memory, can be represented to an arbitrary degree of accuracy by a finite expansion in Volterra series. Considering the expansion of the kernel h_p on an OBF, $\mathfrak{B}_p = \{b_{k,p}\}_{k=0}$, yields:

$$h_p(n_1, \cdots, n_p) = \sum_{k_1=0} \cdots \sum_{k_p=0} g_{k_1,\cdots,k_p} b_{k_1,p}(n_1) \cdots b_{k_p,p}(n_p), \qquad (2)$$

where the Fourier coefficients associated with h_p are given by:

$$g_{k_1,\cdots,k_p} = \sum_{n_1=0} \cdots \sum_{n_p=0} h_p(n_1, \cdots, n_p) b_{k_1,p}(n_1) \cdots b_{k_p,p}(n_p). \qquad (3)$$

By denoting $s_{k_j,p}(n) = \sum_{i=0} b_{k_j,p}(i) u(n-i)$, the input-output relation (1) can be rewritten as:

$$y(n) = \sum_{p=1}^{P} \sum_{k_1=0} \cdots \sum_{k_p=0} g_{k_1,\cdots,k_p} \prod_{j=1}^{p} s_{k_j,p}(n). \qquad (4)$$

The basis functions $\{b_{k,p}(i)\}$ used in this paper are discrete-time Laguerre functions defined by their z-transforms as follows:

$$B_{k,p}(z) = \sqrt{1-\xi_p^2} \frac{z}{z-\xi_p} \left(\frac{1-\xi_p z}{z-\xi_p} \right)^k. \qquad (5)$$

They are characterized by the parameter ξ_p, called the Laguerre pole, and have the two following properties [7]:

$$\begin{aligned} k b_{k,p}(i) &= -\frac{(i+1)\xi_p}{1-\xi_p^2} b_{k,p}(i+1) + \frac{i(1+\xi_p^2)+\xi_p^2}{1-\xi_p^2} b_{k,p}(i) \\ &\quad - \frac{i\xi_p}{1-\xi_p^2} b_{k,p}(i-1), \end{aligned} \qquad (6)$$

$$\frac{\partial b_{k,p}(i)}{\partial \xi_p} = \frac{1}{1-\xi_p^2} \left((k+1) b_{k+1,p}(i) - k b_{k-1,p}(i) \right). \qquad (7)$$

In order to optimize the Laguerre pole associated with the expansion of the kernel h_p, we define the following cost function:

$$J_p = \frac{1}{p \left\| h_p \right\|^2} \sum_{k_1=0} \cdots \sum_{k_p=0} \left(k_1 + \cdots + k_p \right) g_{k_1,\cdots,k_p}^2, \qquad (8)$$

where $\|h_p\|^2 = {}_{n_1=0} \cdots {}_{n_p=0} h_p^2(n_1,\cdots,n_p)$. This cost function is an upper bound of the modeling squared error due to the truncation to a finite order of the Laguerre expansion [3]. By defining:

$$M_{1,l} = \frac{1}{\|h_p\|^2} {}_{n_1=0} \cdots {}_{n_p=0} n_l h_p^2(n_1,\cdots,n_p), \quad l = 1,\cdots,p, \qquad (9)$$

$$M_{2,l} = \frac{1}{\|h_p\|^2} {}_{n_1=0} \cdots {}_{n_p=0} n_l h_p(n_1,\cdots,n_l,n_{l+1},\cdots,n_p) \times h_p(n_1,\cdots,n_l-1,n_{l+1},\cdots,n_p),\ l = 1,\cdots,p \qquad (10)$$

$$Q_{j,p} = \frac{1}{p} {}_{l=1}^{p} M_{j,l}, \quad j = 1,2, \qquad (11)$$

the cost function (8) can be expressed explicitly in terms of the Laguerre pole as follows:

$$J_p = \frac{(1+Q_{1,p})\ {}_p^2 - 2Q_{2,p}\ {}_p + Q_{1,p}}{1 - {}_p^2}. \qquad (12)$$

This result follows the same steps of calculation as in [3]. It is straightforward to show that the function J_p is a pseudo-convex function inside the open convex set $\mathscr{P} = \{\ {}_p \in\ :\ |\ {}_p| < 1\}$. Then any solution to $\frac{J_p}{{}_p} = 0$ is a global minimum of J_p [8]. This minimum is reached for:

$$ {}_{p,opt} = \begin{cases} {}_{o,p} - \sqrt{{}_{0,p}^2 - 1}, & if\ \ {}_{0,p} \geq 1 \\ {}_{o,p} + \sqrt{{}_{0,p}^2 - 1}, & if\ \ {}_{0,p} \leq -1 \end{cases}, \qquad (13)$$

where

$$ {}_{0,p} = \frac{2Q_{1,p}+1}{2Q_{2,p}}. \qquad (14)$$

Let us note that ${}_{0,p}$ is a characteristic of the system since it depends only on $Q_{j,p}$, $j = 1,2$, and therefore on the Volterra kernel h_p.

3. OPTIMAL POLES EXPRESSIONS BASED ON THE LAGUERRE SPECTRA

The optimal pole (13) is related to ${}_{o,p}$ that depends on the Volterra kernel h_p, which means that it is necessary to carry out an estimation of the kernel before determining the optimal pole. In this section, an expression of ${}_{o,p}$ depending on the Laguerre spectrum of the p-th order kernel expanded on any Laguerre basis is investigated. Such an expression will enable us to determine the optimal pole directly from the p-th order estimated Laguerre spectrum without calculating the Volterra kernel coefficients.

Similarly to definitions (9)-(11) associated with the Volterra kernel coefficients, let us define the following quantities that depend on the Laguerre spectrum of the p-th order kernel:

$$T_{1,l} = {}_{k_1=0} \cdots {}_{k_p=0} (2k_l+1) g_{k_1,\cdots,k_p}^2, \quad l = 1,\cdots,p, \qquad (15)$$

$$T_{2,l} = 2 {}_{k_1=0} \cdots {}_{k_{l-1}=0}\, {}_{k_l=1}\, {}_{k_{l+1}=0} \cdots {}_{k_p=0} k_l g_{k_1,\cdots,k_{l-1},k_l,k_{l+1},\cdots,k_p} \times g_{k_1,\cdots,k_{l-1},k_l-1,k_{l+1},\cdots,k_p}, \quad l = 1,\cdots,p \qquad (16)$$

$$R_{j,p} = {}_{l=1}^{p} T_{j,l}, \quad j = 1,2, \qquad (17)$$

Now, the objective is to express $Q_{j,p}, j = 1,2$ as a function of $R_{j,p}$. First of all one can notice that the property (7) combined with formula (3) yields [9]:

$$\frac{}{{}_p} g_{k_1,\cdots,k_p} = \frac{1}{1-{}_p^2} {}_{l=1}^{p} \Big((k_l+1) g_{k_1,\cdots,k_{l-1},k_l+1,k_{l+1}\cdots,k_p} - k_l g_{k_1,\cdots,k_{l-1},k_l-1,k_{l+1},\cdots,k_p} \Big). \qquad (18)$$

Thanks to relation (18) and definitions (15)-(17), we get the following lemma [9]:

Lemma 1 *$R_{1,p}$ and $R_{2,p}$ are linked by means of their derivatives with respect to ${}_p$ as follows:*

$$\frac{R_{1,p}}{{}_p} = \frac{-2}{1-{}_p^2} R_{2,p}, \quad \frac{R_{2,p}}{{}_p} = \frac{-2}{1-{}_p^2} R_{1,p}. \qquad (19)$$

The orthonormality of the Laguerre basis allows to get the following relation: $\|h_p\|^2 = {}_{k_1=0} \cdots {}_{k_p=0} g_{k_1,\cdots,k_p}^2$. Then, from the definitions of $R_{1,p}$ and of J_p, a simple calculation yields

$$R_{1,p} = 2p \|h_p\|^2 J_p + p \|h_p\|^2. \qquad (20)$$

Thus

$$\frac{R_{1,p}}{{}_p} = 2p \|h_p\|^2 \frac{J_p}{{}_p}. \qquad (21)$$

By using the relations (12) and (20), and the lemma 1, we get:

Lemma 2 *The terms $Q_{j,p}$ associated with the Volterra kernel h_p and $R_{j,p}$ associated with the corresponding Laguerre spectrum are linked by:*

$$R_{1,p} = \frac{p\|h_p\|^2}{1-{}_p^2} \left(2\left(1+{}_p^2\right) Q_{1,p} - 4\ {}_p Q_{2,p} + 1 + {}_p^2 \right), \qquad (22)$$

$$R_{2,p} = \frac{2p\|h_p\|^2}{1-{}_p^2} \left(-2\ {}_p Q_{1,p} + \left(1+{}_p^2\right) Q_{2,p} - {}_p \right). \qquad (23)$$

By solving the system of equations (22) and (23), we get the expressions of $Q_{j,p}$ as a function of $R_{j,p}$, $j = 1,2$. Then, by replacing these expressions in (14), ${}_{o,p}$ can be written as follows:

$$ {}_{o,p} = \frac{(1+{}_p^2) R_{1,p} + 2\ {}_p R_{2,p}}{2\ {}_p R_{1,p} + (1+{}_p^2) R_{2,p}}. \qquad (24)$$

This result is summarized by the following theorem:

Theorem 1 *The optimal pole of the Laguerre basis, associated with the expansion of the Volterra kernel h_p, is obtained from the Laguerre spectrum, associated with the expansion of the same kernel on an arbitrary Laguerre basis characterized by the pole ${}_p$, as follows:*

$$ {}_{p,opt} = \begin{cases} {}_{o,p} - \sqrt{{}_{o,p}^2 - 1}, & if\ \ {}_{o,p} \geq 1 \\ {}_{o,p} + \sqrt{{}_{0,p}^2 - 1}, & if\ \ {}_{0,p} \leq -1 \end{cases}, \qquad (25)$$

where ${}_{0,p}$ is given by the formula (24).

This result is particularly meaningful. Equation (25) means that the optimal Laguerre pole ${}_{p,opt}$ is obtained from ${}_{o,p}$ that can be recovered in using the Laguerre spectrum associated with an arbitrary pole ${}_p$, as stated by formula (24) with equations (15)-(17) for the calculation of $R_{1,p}$ and $R_{2,p}$.

4. ADAPTIVE ESTIMATION METHOD FOR LAGUERRE-VOLTERRA FILTERS

In practical case the expansion of the Volterra kernels on a Laguerre basis is truncated to a finite order K. Consequently, for a given Laguerre pole ${}_{p}$, the value of ${}_{0,p}$ calculated in using truncated expressions of (15) and (16) is only an approximation of the actual characteristic of the system. This approximation depends on the Laguerre pole and on the Laguerre spectrum, via $R_{1,p}$ and $R_{2,p}$. For a fixed pole ${}_{p}$, the optimization of the Laguerre spectrum improves the approximation of ${}_{o,p}$. It is then possible to determine the optimal pole corresponding to the current approximation of ${}_{o,p}$. This process is iterated until convergence.
Let us consider the LVF described as follows:

$$y(n) = \sum_{p=1}^{P}\sum_{k_1=0}^{K-1}\cdots\sum_{k_p=0}^{K-1} g_{k_1,\cdots,k_p}\prod_{j=1}^{p} s_{k_j,p}(n) = G^T S(n),$$

where $G = \left(G_1^T \cdots G_p^T \cdots G_P^T\right)^T$ and $S(n) = \left(S_1^T(n) \cdots S_p^T(n) \cdots S_P^T(n)\right)^T$, G_p containing the Fourier coefficients $g_{k_1,\cdots,k_p}$ and $S_p(n)$ the p-th order cross-products of the $s_{k,p}(n)$ signals.
The proposed method can be viewed as a block-NLMS type method. Its principle is to adapt Fourier coefficients until a given convergence criterion be satisfied, then the Laguerre poles are estimated, and the calculation is iterated with new data. Note that the Fourier coefficients are adapted by means of the NLMS algorithm (26) while the Laguerre poles are estimated in using (25).

$$G(n+1) = G(n) + \frac{}{a + \|S(n)\|^2}\left(d(n) - G^T(n)S(n)\right)S(n). \quad (26)$$

a is a small positive constant, is the step-size and $d(n)$ is the actual output of the system to be modeled. The estimation of a new Laguerre pole occurs when the estimated Fourier coefficients are nearly constant during a given time window of length N_O. The adaptive procedure is summarized as follows:

Parameters:
- K: truncation order.
- : step-size ($0 < \ < 2$).
- a: small positive constant.
- ${}_{p}$: convergence threshold.
- N_O: window length.

Initialization:
- Set the vector $G(0)$ equal to zero and $n = 0$.
- Set arbitrary initial values for ${}_{p}$, $p = 1, \cdots, P$ and build the associated Laguerre bases.

Computation:
1. Calculate the filtered inputs $s_{k,p}(n)$ associated with the Laguerre filters $B_{k,p}(z)$ defined by (5), organize the cross products of the filtered inputs into the vectors $S_p(n)$ to generate the vector $S(n)$.
2. Estimate the Laguerre spectra in using (26).
3. For $p = 1 \cdots, P$:
   - If $\|G_p(n+1) - G_p(n)\|^2 < {}_{p}$ during the interval $[n+2-N_O, n+1]$:
     (a) Calculate $R_{1,p,n}$ and $R_{2,p,n}$ in using truncated expressions of (15)-(16).
     (b) Evaluate ${}_{o,p,n}$ in using (24).
     (c) Determine new Laguerre poles according to (25).
     (d) Build the associated Laguerre bases, increment n and return to step 1.
   - Else increment n and return to step 1.

5. SIMULATION RESULTS

To evaluate the performance of the proposed method, we present simulation results obtained for two identification experiments. For both experiments, the measurement noise was assumed to be a white Gaussian process with zero mean and a variance such that the Signal-to-Noise Ratio (SNR) be equal to 30 dB. All the simulation results were obtained as ensemble averages over 50 independent runs.
The identification of a second order Volterra system described below was first considered.

First order kernel :

$$H_1(z) = \frac{z(z+0.5)}{(z-0.3)(z-0.2)}.$$

Second order kernel :

$$h_2(i,j) = 0.25h(i)h(j), \quad h(i) = \mathscr{Z}^{-1}\left\{\frac{z(z+1)}{(z-0.8)(z+0.8)}\right\}.$$

This system was simulated as a Volterra system with memory $M = 20$. By taking the symmetry of the quadratic kernel into account, this filter has 230 parameters to estimate. The input signal was white, Gaussian, centered with an unit variance. To validate the theoretical analysis presented in the previous sections and to evaluate the quality of the estimated Laguerre poles, an off-line calculation of the cost functions J_1 and J_2 defined in (8) was done. By plotting these functions, it appears that each function has a single minimum respectively located at ${}_{1,opt} = 0.525$ and ${}_{2,opt} = 0.733$.
In order to fill the requirements of [3], a unit delay was incorporated in the transfer functions defining the above system. The method of [3] provided the following poles ${}_{1,cam} = 0.525$ and ${}_{2,cam} = 0.734$. These values are similar to the optimal values ${}_{1,opt}$ and ${}_{2,opt}$ and are to be compared with those provided by the proposed estimation method.
The Laguerre poles was initialized to 0. The Volterra kernels expansions on Laguerre bases were truncated to $K = 7$; thus the resulting LVF has only 35 Fourier coefficients to estimate while the standard Volterra filter has 230 parameters. The step-size of the NLMS algorithm was chosen as $= 0.3$. The convergence thresholds were chosen equal to ${}_{p} = 10^{-3}$, $p = 1,2$ and the window length was $N_O = 50$.

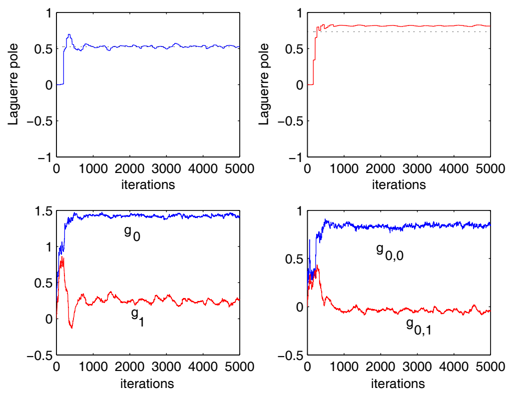

Figure 1: Estimated Laguerre poles (Top) and two estimated Fourier coefficients (Bottom) corresponding to the linear (Left) and quadratic (Right) kernels

From figure 1 one can conclude that the poles converge towards values close to the poles obtained with the Campello's method and also close to the optimal values ${}_{1,opt}$ and ${}_{2,opt}$ (dotted lines on Fig.1).
As previously stated the advantage of the proposed method is that

the *a priori* knowledge of the Volterra kernels is not needed, and both Fourier coefficients and Laguerre poles are simultaneously estimated. However, one important question is that of the choice of the convergence threshold $_p$ and of the window length N_0. When the value of $_p$ is chosen too small, a huge number of iterations can be necessary for the convergence of a new estimated Laguerre pole. Consequently, a particular care should be taken for selecting these parameters. To illustrate the convergence of the overall identification procedure, the Mean Square Error (MSE) associated with the proposed adaptive algorithm was compared with those obtained when the Laguerre poles were arbitrarily chosen equal to 0.2 and when the Laguerre poles were chosen equal to their optimal values $_{1,opt}$ and $_{2,opt}$ (Fig. 2). In steady-state, the performances of the proposed algorithm are similar to those obtained with the LVF the poles of which are optimal, but the convergence is slower. For the LVF with arbitrary poles, to obtain a similar performance, the truncation order had to be increased ($K = 9$).

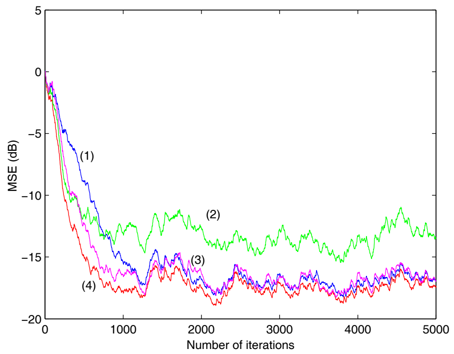

Figure 2: Comparison of four configurations of LVFs:(1)- poles optimized with the proposed adaptive algorithm, (2)- poles arbitrarily fixed and $K = 7$, (3)- poles arbitrarily fixed and $K = 9$, (4)- poles fixed at their optimal values

The second simulated system was a simplified model of a Power Amplifier (PA) with memory that is an important device in radio communication systems. A relatively simple baseband behavioral model that accommodates memory as well as nonlinear behavior is the Wiener model, i.e. a linear filter followed by a memoryless nonlinearity given by [10]: $A(r) = \frac{2r}{1+r^2}$, $A(r)$ and r being the output and input signal amplitudes of the memoryless PA respectively. The linear filter is a low-pass fourth-order Butterworth filter with a cutoff frequency of 0.1 cycles/sample. We considered the transmission of a 8-PAM signal through an AWGN (Additive White Gaussian Noise) channel with SNR=30 dB. The PA with memory, to be identified, was modeled as a linear-cubic Volterra system. From Figure 3a we can conclude that the Laguerre poles respectively associated with the linear and the cubic kernels converge to the same value. This behavior was predictable since the dominant dynamics associated with the linear and the cubic kernels are the same for a Wiener model. As for the first example, Figure 3b compares three configurations for representing the PA with memory: a LVF truncated at $K = 9$ driven by the proposed method, a standard Volterra filter with memory $M = 16$, and a LVF truncated at $K = 10$ with a pole arbitrarily chosen equal to 0.25. The truncation orders and the memory were selected so that the steady-state performances are similar. For the proposed method, the convergence thresholds were chosen such as $_p = 10^{-3}$, $p = 1, 3$ and $N_O = 75$. The parameters in each configuration were adapted by using the NLMS algorithm with a step-size equal to 1.0. Both configurations gave comparable performances but with a huge difference in terms of the parametric complexity. By taking triangular representations of the kernels into account, the LVF with optimized pole has 174 parameters whereas the LVF with arbitrary poles and the Standard Volterra filter have respectively 230 and 832 parameters. Hence, a significant parametric complexity reduction is achieved with the optimized LVF while providing similar

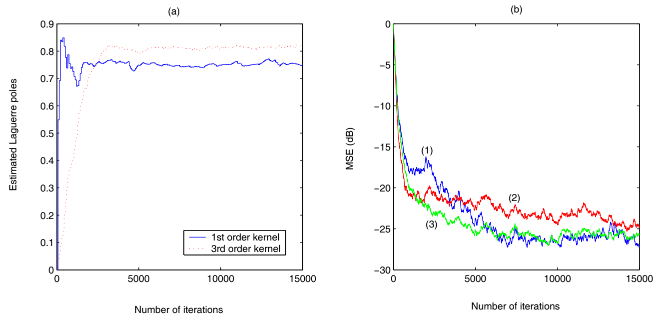

Figure 3: (a)- Laguerre poles estimated with the adaptive algorithm (b)-Comparison of three configurations: (1)- LVF using the proposed method, (2)- Standard Volterra filter, (3)- LVF with poles arbitrarily fixed at 0.25.

performances in terms of MSE.

6. CONCLUSION

In this paper, an analytical solution to the optimization of Volterra kernels expansions on Laguerre bases has been presented. This solution has been expressed in terms of the estimated Laguerre spectrum associated with each Volterra kernel. Then an adaptive method has been proposed to optimize both Laguerre spectra and Laguerre poles. The performance of the proposed identification method have been illustrated by means of simulation results that show the usefulness of the proposed method. In future works the authors intend to investigate analytical solution to the optimization problem of others OBFs for Volterra kernels expansions.

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