# THE OPTIMAL DIRECT DESIGN OF BANDPASS WAVE DIGITAL LADDER FILTERS FOR DISTINCTLY SPECIFIED AMPLITUDES IN THE TWO STOPBANDS

Mohamed Yaseen
Dept. of Electrical & Electronic Eng., Faculty of Eng., University of Assiut, Assiut, EGYPT
Tel : 088-334688, Fax: 088-332553, Email: M_yaseen3@yahoo.com

**ABSTRACT**

The paper presents an optimal and direct complete design method for bandpass wave digital filters (WDFs) having ladder structures and approximating different minimum losses in the two stopbands. Moreover, the resulting loss characteristic is restricted to be equiripple in the three bands, i.e., it has the same features of elliptic response. The approximation is carried out directly without applying frequency transformation techniques. It relies on applying interpolation techniques combined with the Remez-exchange algorithm . The resulting transmission function is synthesized by successive partial extraction of the poles at zero and infinite reference frequencies from the successive resulting impedance functions followed by successive extraction of the finite transmission zeros from the remaining admittance functions. The wave digital realization is finally obtained by applying three-port series and parallel adaptors.

## 1. INTRODUCTION

Bandpass digital filters are required in many communication systems and applications. Previously [1-4], these filters were obtained through applying design methods based on frequency-transformation techniques. However, the design methods based on frequency-transformation techniques suffer from many disadvantages: 1- Computational complexity. 2- Degree non optimality, which results from two sources. On one hand, the degree of the filter must be even. On the other hand, there is no possibility to design two stopbands with two different minimum losses. 3- The number of transmission zeros must be divided equally between the the two stopbands .

Consequentlty, the direct design of bandpass digital filters is the optimal solution because it overcomes the above disadvantages. Recently, effective approximation methods [5-7] have been presented for bandpass WDFs having lattice structures.

Now, it is useful and interesting to apply the concept of direct design for bandpass WDFs having ladder structures. In Ref. [8], the direct design of bandpass WDFs having simple adder structures has been considered. For these structures, the transmission function exhibits all of its zeros at zero and infinite reference frequencies. Consequently, the resulting amplitude characteristic has the features of Chebyshev response, i.e., it ripples equally in the passband and decreases monotonically in the two stopbands.

In this contribution, the direct design of bandpass WDFs having optimal ladder structures is considered. For these structures, the transmission function is formulated such that it has transmission zeros at finite frequencies beside the zero and infinite reference frequencies. This means that the resulting amplitude characteristic will have the features of Elliptic response, i.e., it ripples equally in the passband and the two stopbands.

## 2. THE APPROXIMATION PROBLEM

Let us be given bandpass amplitude specifications in the digital domain. It is required to approximate these specifications directly with a bandpass wave digital ladder structure exhibiting equiripple response in the three bands. In reference domain, the corresponding transmission function for even degree bandpass ladder structure is :

$$S_{21}(\psi) = \frac{f(\psi)}{g(\psi)} \tag{1}$$

where $\psi = \Sigma + j\phi$ is the complex frequency variable in the reference domain. The polynomial $g(\psi)$ is strictly Hurwitz with degree n. On the other hand , $f(\psi)$ is restricted to possess all of its roots on the imaginary axis. For a bandpass ladder structure, the transmission function of Eq. (1) is formulated as:

$$S_{21}(\psi) = \frac{\psi \prod_{i=1}^{m/2} (\psi^2+\beta_i^2)}{\sum_{k=0}^{n} b_k \psi^k} \tag{2}$$

Where n is the filter degree and m = n-2 Accordingly, transmission zeros are generated at zero and infinite frequencies. This is necessary for getting regular and non-complicated structures.
The transmission function can be formulated as:

$$S_{21}(\psi)=\frac{\psi\sum_{i=o}^{m/2} a_i\psi^{2i}}{\sum_{k=o}^{n} b_k\psi^k}, \quad a(m/2)=1 \qquad (3)$$

Consequently, the squared amplitude function is:

$$|S_{21}(j\phi)|^2 = S_{21}(\psi)S_{21}(-\psi) \quad ,\psi=j\phi$$

$$=\frac{-\psi^2\sum_{i=o}^{m} c_i\psi^{2i}}{\sum_{k=o}^{n} d_k\psi^{2k}},\psi=j\phi, c_m=1 \qquad (4)$$

$$=\frac{R(\psi)}{Q(\psi)} \quad ,\psi=j\phi$$

where:

$$R(\psi)=f(\psi)f(-\psi) \text{ and } Q(\psi)=g(\psi)g(-\psi) \quad (5)$$

The restriction here is that the polynomial $R(\psi)$ must have all of its zeros on the imaginary axis and with even multiplicity. This insures that the property of the polynomial $f(\psi)$ will be reserved.

## 3. THE APPROXIMATION PROCEDURE

Now, the direct approximation procedure for direct generation of a bandpass structure exhibiting equiripple response in the three bands is summarized:

1- Translate the given loss specifications into corresponding specifications for the squared amplitude function.

2- Set initial degree n for the filter. An optimal value for the degree can be determined by using the same procedure given for bandpass lattice structures [6].

3- At a set of frequencies (n+m+1 points), interpolate the squared amplitude function for the coefficients c and d. This is achieved by spreading the interpolation points properly over the passband and the two stopbands, with the band edges held as fixed points. Within each band, the interpolation points can be initially distributed such that in the digital domain , they become in an equidistant arrangement. This has been detected to be sufficient for the convergence process. Note that the digital frequencies are reflected into the reference domain through the bilinear transformation:

$$\psi=\frac{z-1}{z+1} \quad ,z=e^{pT} \quad , \quad \phi=\tan(\omega T/2) \quad (6)$$

where $p=\sigma+j\omega$, is the complex frequency variable in the digital domain and T is the sampling period.

4- Apply the Remez-exchange algorithm to change the set of interpolation points for optimal coefficient values.

5- Get the resulting amplitude response and test. If it satisfies the given specifications within reasonable reserve margins, stop. If it over- satisfies or under-satisfies the given specifications, decrease or increase (respectively) the filter degree and go to step 3.

6- After obtaining the optimal transmission function, synthesize it to get the reference structure as follows.

### 4. THE SYNTHESIS PROBLEM

The synthesis of the resulting transmission function is summarized:

1- From the resulting transmission function $S_{21}(\psi)$, determine the corresponding reflection function:

$$S_{11}(\psi)=\frac{h(\psi)}{g(\psi)} \qquad (7)$$

where due to losslessness [9-10], the following relationship holds:

$$h(\psi)h(-\psi)=g(\psi)g(-\psi)-f(\psi)f(-\psi) \qquad (8)$$

2- From the resulting reflection function, and assuming unity input reference resistor, the corresponding input impedance is determied:

$$Z_i(\psi)=\frac{1+S_{11}(\psi)}{1-S_{11}(\psi)} \qquad (9)$$

3- Now, the realization is relying on the partial extraction of the poles at infinite and zero reference frequencies from the successive resulting impedance functions, followed by the extraction of the corresponding transmission zeros from the remaining admittance functions. In case of partial extraction of a pole at infinite frequency, the value of the extracted inductor is determined according to:

$$L=\min\left[\frac{Z(j\phi_{s21})}{j\phi_{s21}},\frac{Z(j\phi_{s22})}{j\phi_{s22}},..,\frac{Z(j\phi_{s2k})}{j\phi_{s2k}}, \lim_{\psi=\infty}\psi Z(\psi)\right] \qquad (10)$$

where $\phi_{s21},\phi_{s22},.....,\phi_{s2k}$ are the transmission zeros in the second stopband. On the other hand, in case of partial extraction of a pole at zero frequency, the value of the extracted capacitor is determined according to:

$$\frac{1}{C}=\min\left[Z(j\phi_{s11})j\phi_{s11}, Z(j\phi_{s12})j\phi_{s12},.., Z(j\phi_{s1q})j\phi_{s1q}, \lim_{\psi=0}\psi Z(\psi)\right] \qquad (11)$$

where $\phi_{s11},\phi_{s12},....,\phi_{s1q}$ are the transmission zeros in the first stopband.

4- After extracting a pole at infinite or zero frequency from the impedance function, a transmission zero is extracted from the remaining admittance function in form of shunt arm composed of an inductor and capacitor in series. The procedure is repeated until the complete reference structure is obtained.
5- Finally, the wave digital realization is obtained by applying 3-port series and parallel adaptors.

**5. DESIGN EXAMPLE**

Now, let us apply the above method through a design example. Considering bandpass loss specifications given as:

The first stopband extends from 0 to 3 kHz with min. loss =35 dB

The passband extends from 4.5 to 7.5 kHz with max. loss = 0.5 dB.

The second stopband extends from 9 to 12.5 kHz with min. loss =30 dB.

The sampling frequency is 25 kHz.

These specifications have been approximated by a wave digital ladder structure with degree =6. The following final interpolation values were reached after 6 iterations:

| **Freq. kHz** | **Amplitude** |
|---|---|
| 1.8 | 0.01778279 |
| 2.74004216268 | 0.0 |
| 3.0 | 0.01778279 |
| 4.5 | 0.9332543 |
| 4.7008990544 | 1.0 |
| 5.346875 | 0.9332543 |
| 6.39294519129 | 1.0 |
| 7.5 | 0.9332543 |
| 9.0 | 0.03162277 |
| 9.2566762390046 | 0.0 |
| 10.271875 | 0.03162277 |

Accordingly, the following results are available:

$$g(\psi)=\sqrt{45.757109398}(\psi^6+1.225864768\psi^5+4.170057714\psi^4+2.909608277\psi^3+4.10266041\psi^2+1.120069474\psi+0.941320079)$$

$$h(\psi)=\sqrt{45.757109398}(\psi^6+0.291793227\psi^5+3.472186252\psi^4+0.444762979\psi^3+3.452375403\psi^2+.141005489\psi+0.941320958)$$

The resulting loss response is shown in Fig. 1. Assuming unity input reference resistor, the resulting input impedance is:

$$Z_i(\psi)=\frac{N(\psi)}{D(\psi)}$$

where:

$$N(\psi)=g(\psi)+h(\psi) \text{ and } D(\psi)=g(\psi)-h(\psi)$$

$$N(\psi)=2.0\psi^6+1.517658\psi^5+7.642244\psi^4+3.354371\psi^3+7.555041\psi^2+1.261075\psi+1.882641$$

$$D(\psi)=0.934971541\psi^5+0.697871461\psi^4+2.464845298\psi^3+0.650290638\psi^2+0.979063984\psi$$

This input impedance has been synthesized according the procedure given in the text. The resulting reference structure is shown by Fig. 1. The corresponding element values are:

$L_1 = 1.6235201$ $L_2 = 0.591044247$
$C_2 = 0.315376997$ $C_3 = 0.253672272$
$L_4 = 8.1697425842$ $C_4 = 0.9518180$
$L_5 = 8.5100560$ $R_5 = 6.7699070$
$C_5 = 0.1077918$

The wave digital realization is obtained by applying the three-port series and parallel adaptors with the realizability conditions be respected [4].

**REFERENCES**


[1] A. Fettweis, Transmultiplexers with Either Analog Conversion Circuits, Wave Digital Filters, or SC-Filters- a Review , IEEE Trans. Commun. vol. COM-30, pp. 1598-1613, July 1982.
[2] A. Fettweis, Principles for Multiplier-Free Single-Way PCM-FDM and Audio / FDM Conversion , Arch. Elek. Uberrtragung, vol. 32, pp 441-446, Nov. 1978.
[3] A. Fettweis, Multiplier-Free Modulation Schemes for PCM / FDM and Audio / FDM Conversion . Arch. Elek. Uberrtragung, vol. 32, pp. 477 -485, Dec. 1978.
[4] A. Fettweis, Wave Digital Filters, Theory and Practice, Proc. of IEEE , vol. 74, pp. 270-327, Feb. 1986.
[5] M.. Yaseen and T. Henk, Direct Design Method for Bandpass Wave Digital Lattice Filters with Arbitrary Loss Specifications , Proc. of the 2-nd International Symposium on Signals, Systems and Electronics I SSSE-92 , vol.. 2, pp. 470-473, Paris, France, Sept. 1-4, 1992.
[6] M. Yaseen, The Optimal and Straightforward Design

of Bandpass Wave Digital Lattice Filters with Arbitrary Loss Specifications , the International Journal of Circuit Theory and Applications, vol. 25, pp. 209-218, 1997.
[7] M. Yaseen, Robust Simultaneous Amplitude and Phase Approximations Oriented to Bandpass wave Digital Lattice Filters , the International Journal of Circuit Theory and Applications, vol.. 26, pp. 179-189, 1998 .
[8] M. Yaseen, Direct Design of Bandpass Wave Digital Ladder Filters , Proc. of the European Conference of Signal Processing EUSIPCO-2000, vol. IV, pp. 2545-2548, Tampere, Finland, Sept. 4-8, 2000.
[9] V. Belevitch, Classical Network Theory, Holden-Day, 1968.
[10] J. D. Rhodes, Theory of Electrical Filters, Wiley, 1976.

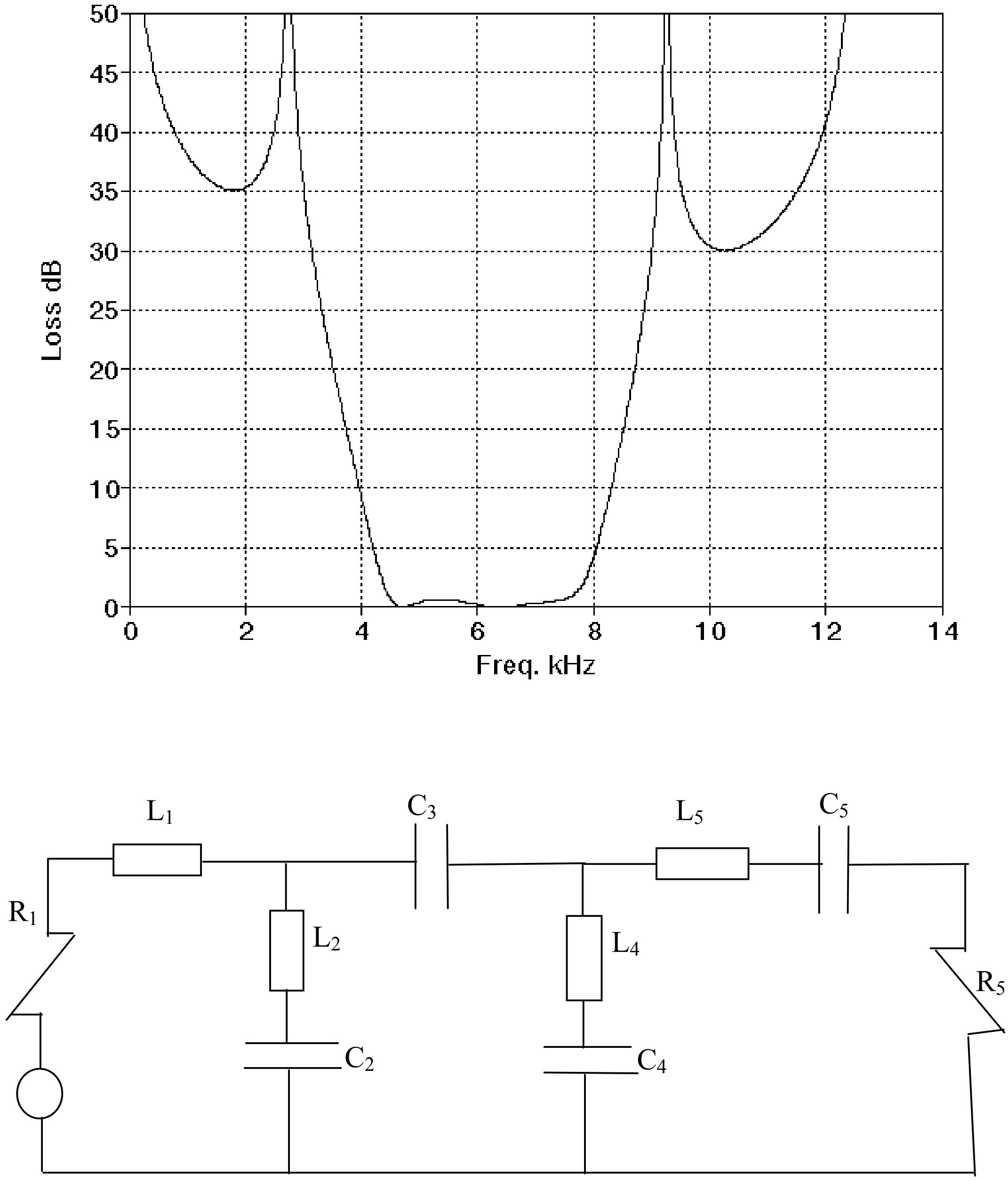


Fig. 1 The loss response and the reference structure