Multi-Sensor Based Ambient Assisted Living System

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M.Sc. Defense by
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Outline

- Objective
  - Motivation
  - Previous Works
- Hardware Implementations
- Signal Analysis Methods
  - Detection Algorithms
- Experimental Results
- Conclusion
Part – I : Living Alone Safely

- Objective
- Motivation
- Previous works
Emergency Situation Detection Systems

- Falling person detection
- Human footstep detection
- Unusual inactivity detection
- Indoor flooding detection
- Uncontrolled flame detection
- Gas leak detection
- Human motion detection
- Pet movement detection

New studies

Past studies
UNCONTROLLED FLAME ALARM!

FALLING PERSON ALARM!
What is our motivation?

- Number of the elderly people increase rapidly.
- About one-third of people over 65 falls unexpectedly [1].
- Falling is the most common cause of injury amongst seniors.
- Fall related health and injury cost is considered as billions of dollars worldwide.
- Unusual inactivity detection is also very important because of the sudden diseases like heart attacks.

Wearable Fall Detection Systems:

- 2007 Wearable sensors
- 2013

Ambient Assisted Living Systems for Fall Detection:

- 2004 Computer vision based systems
- 2011 Kinect sensors
- 2013
Wearable Systems

- Commercially available
- Only one sensor
- No installation cost
- Independent from the environment

Disadvantage:
- A necessity of carrying sensors always

Training the system for many different situations is possible.

Disadvantages:
- Privacy enemy
- High computational power of processors
- Large number of cameras to handle blind spots
- Installation cost
- High cost of the overall system
Kinect Sensor Based Systems

- Almost the same conditions with a computer vision based system
- Less privacy enemy than the computer vision based system but not a privacy-friendly system

Our Multi-Sensor Based System

- Privacy-friendly
- Unnoticeable system to the person
- Lower cost than camera and Kinect based systems
- Only 1-D signal processing
- No need to consider forgetfulness

Disadvantages:
- Large number of sensors
- Installation cost
Part – II : Hardware Implementations

- Vibration sensor
- PIR sensor
- Arduino microprocessor
Vibration Sensor

GS-20DX (~$25)

Vibration Sensor

- Sampling at a rate of 500 Hz and digitized with 8-bit resolution
Passive Infrared (PIR) Sensor

Paradox 476+ (~$5)
Two sensing elements eliminate the noise caused by vibration, temperature changes and sunlight.

Heat source must pass across the sensor in a horizontal direction.

Passive Infrared (PIR) Sensor

- Infrared transmission range of 8 to 14 µm
Passive Infrared (PIR) Sensor

• Sampling at a rate of 100 Hz and digitized with 8-bit resolution
Arduino Uno32 Prototyping Platform

- PIC32MX320F128 processor
- 80 Mhz 32-bit MIPS
- 128K Flash, 16K SRAM
- 42 available I/O
- 12 analog inputs
- 3.3V operating voltage
- 75mA typical operating current
Auto-Dial Alarm System

Alarm trigger

Arduino Uno32

HT9200A DTMF tone generator & amplifier circuit

Isolation transformer

+12V

Circuit to open/close the telephone lines

Relay

Telephone lines

DTMF tones
Part – III : Signal Analysis Methods

- Discrete Fourier transform
- Discrete wavelet transform
- Complex wavelet transform
• The main purpose is to extract meaningful information from the one-dimensional signals.
• Example application: Falling person detection using the vibration sensor.
• Frequency contents of the one-dimensional signals are analyzed in eight sub-bands.
• More emphasis is given to lower frequencies by assigning more sub-bands to them.
• Energies of these frequency sub-bands are employed as eight feature parameters.
• Boundary frequencies: 1.95 Hz, 3.91 Hz, 7.81 Hz, 15.63 Hz, 31.25 Hz, 62.5 Hz, 125 Hz, and 250 Hz.
where $X(k)$ is the DFT of the input signal and $G(m)$ is the energy values of the eight frequency sub-bands. $B(m)$ represents the indices of the sub-band boundaries. Energy values are employed as eight feature parameters.
Mel-Frequency Cepstral Coefficients (MFCC)

\[
G(m) = \sum_{k \in B(m)} |X(k)|^2, \quad m = 1, \ldots, 8
\]

- Mel-frequency cepstral coefficients, \( C(u) \), are calculated with the following formula using the discrete cosine transform (DCT):

\[
C(u) = DCT^{-1}\{\log_{10}(G(m))}\}, \quad u = 1, \ldots, 8
\]
Seven-level discrete wavelet transform is applied to the input signal using one of the wavelets: Haar, Daubechies–2, Daubechies–4, or Biorthogonal–3.3.
Dual-Tree Complex Wavelet Transform (DT-CWT)
Dual-Tree Complex Wavelet Transform (DT-CWT)

• Problems with Real Wavelets:
  – Oscillations
  – Shift-Variance
  – Aliasing

• The Fourier transform does not suffer from them.
• An another solution: Complex Wavelets [6]
• The redundancy factor of a $d$-dimensional signal: $2^d$

• Using $u_1[n]$ and $u_2[n]$ in a sequentially switched manner for a low-pass filtering of the input signal, we constructed a time-varying single-tree lifting structure that keeps the benefits of DT-CWT.
Single-Tree Complex Wavelet Transform (ST-CWT)

\[ \tilde{H}_1(z) = \frac{1}{2} + z^{-1}U_1(z) \]

\[ \tilde{H}_2(z) = \frac{1}{2} + z^{-1}U_2(z) \]

- Filters \( U_1(z) \) and \( U_2(z) \), or equivalently \( \tilde{H}_1(z) \) and \( \tilde{H}_2(z) \) are designed using the following constraints:
(i) Since $\tilde{h}_i[n]$ is a half-band filter, $\tilde{h}_i[2n] = 0$ for $n \neq 0, i = 1, 2$, for perfect reconstruction in a lifting structure.

(ii) Filters $\tilde{h}_1[n]$ and $\tilde{h}_2[n]$ must have approximate group delays of 1/4 and 3/4, respectively so that there exist 0.5 delay difference between the two filters [7].

(iii) Filters $\tilde{H}_1(z)$ and $\tilde{H}_2(z)$ must have a zero at $z = -1$, that is, $\sum_n \tilde{h}_i[n](-1)^n = 0$ for $i = 1, 2$.

Shift-Invariance Property Based Comparison

- A unit step signal and its shifted versions are given as input to the wavelet filter-banks.
- Energies of the wavelet coefficients at the third level decomposition are calculated.
• Haar and Daubechies-2 wavelet filter-banks.
• Daubechies-4 and Biorthogonal-3.3 wavelet filter-banks.
- Dual-tree and single-tree complex wavelets.
The exact number of real multiplications. $N = 1024$ is the number of signal samples in each window; $M = 8$ is the number of the frequency sub-bands; $p$ is the number of nonzero filter coefficients, and $r = 7$ is the number of the levels in a wavelet-tree.
Part – IV : Detection Algorithms

• Data collection
• AAL system
• Experimental results
Data Collection

• Vibration sensor signal records are taken in different experimental environments:
  – Third floor of a new building (concrete floor),
  – Second floor of an old building (hardwood floor),
  – Fourth floor of a new building (hardwood floor).

• Vibration sensor signals may contain various frequency components depending on
  – the architecture of the building,
  – running machines in the building,
  – the person living in the house.
Datasets

• Five different datasets are employed to train and test the detection systems:
  – Motion-inactivity dataset -> One PIR sensor
  – Footstep dataset -> One vibration sensor
  – Falling dataset-1 -> One vibration sensor
  – Falling dataset-2 -> One vibration sensor, two PIR sensors
  – Flooding dataset -> One vibration sensor

• Two-second-long signal windows are used in the detection algorithms.
Human Motion Detection

1. Start
2. Read PIR sensor data
3. Calculate variance
4. If variance > threshold, decide motion activity
5. If variance ≤ threshold, repeat from Step 2
Unusual Inactivity Detection

1. **START**
2. Read PIR sensor data
3. Calculate variance
4. **Decide unusual inactivity situation**
   - **NO**
     - **Motion?**
       - **NO**
         - **Variance > Threshold**
           - **YES**
             - **Decide instant motion**
           - **NO**
             - **Measure how long there is no motion**
               - **YES**
                 - **Inactivity situation?**
                   - **YES**
                     - **Warn the person in the house and monitor the person one more minute**
                   - **NO**
                     - **Inactivity situation?**
                       - **YES**
                         - **Warn the person in the house and monitor the person one more minute**
                       - **NO**
                         - **Inactivity situation?**
Experimental Results

• The time period is taken 10 minutes to decide the unusual inactivity situation in the tests.

• 20-hours unusual inactivity record is divided into 10-minutes parts and totally 120 parts are tested.

• 100% of the 120 parts are classified correctly.
Human Footstep Detection

- One adaptive threshold, $T_a$, is used.
- Three-state Markov models are trained for each of the two classes.
- To classify a two-second-long signal, $x$, three states are determined by:

  - $if$ $-T_a < x[n] < T_a$, State-0
  - $if$ $x[n] > T_a$, State-1
  - $if$ $x[n] < -T_a$, State-2
Adaptive Threshold

Variation in the sensor signal, $\sigma_k^2$, originated from the external factors are observed by changing the background variance, $\sigma_{k,b}^2$, by:

$$
if \quad \sigma_k^2 > \beta \sigma_{k-1,b}^2, \quad \sigma_{k,b}^2 = \sigma_{k-1,b}^2
$$
$$
everse, \quad \sigma_{k,b}^2 = \alpha \sigma_{k-1,b}^2 + (1 - \alpha) \sigma_k^2
$$
Adaptive Threshold

• Adaptive threshold value, $T_a$, is updated using the following equation:

$$T_a = T_a \frac{\sigma_{k,b}^2}{\sigma_{k-1,b}^2}$$

• Threshold should not be updated when there is a walking person in the environment.
Human Footstep Detection

1. START
2. Read vibration sensor data
3. Calculate transition matrix
4. MM classifier
5. Footstep? (YES/NO)
   - YES: Record as a statistics for the daily activity reports
   - NO: Update threshold for the MM classifier
## Experimental Results

### Human Footstep Detection System (Testing Dataset)

<table>
<thead>
<tr>
<th>Actual Class</th>
<th>Predicted Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Sources</td>
</tr>
<tr>
<td>Other Sources</td>
<td>109</td>
</tr>
<tr>
<td>Human Footstep</td>
<td>0</td>
</tr>
</tbody>
</table>

### Human Footstep Detection System (All Dataset)

<table>
<thead>
<tr>
<th>Actual Class</th>
<th>Predicted Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Sources</td>
</tr>
<tr>
<td>Other Sources</td>
<td>199</td>
</tr>
<tr>
<td>Human Footstep</td>
<td>0</td>
</tr>
</tbody>
</table>
Falling Person Detection

- Falling dataset-1 [8]
  - Vibration sensor based detection algorithm

- Falling dataset-2 [9]
  - Vibration sensor based detection algorithm
  - Two-PIR-sensor based detection algorithm
  - Multi-sensor based detection algorithm


Fall Detection Using Vibration Sensor

• A typical instant falling event generally lasts about two seconds.
• Frequency content of the vibration sensor signal is analyzed in eight sub-bands.
• Energies of these frequency sub-bands are employed as eight feature parameters.
• DFT, MFCC, DWT, DT-CWT, and ST-CWT signal analysis methods compared to each other.
• SVM classifiers are trained and used.
Fall Detection Using Vibration Sensor

1. **START**
2. Read vibration sensor data
3. Find maximum and minimum values
   - (max - min) > Threshold
     - NO
     - YES: Frequency analysis and feature extraction
4. Human footstep detection system
5. Classification
6. Falling person?
   - NO
   - YES: Decide there is a falling person
Experimental Results

Numbers of "true detection" versus "false alarm" for 1024-sample-long signal windows, using the SVM classifier.

<table>
<thead>
<tr>
<th>Feature extraction methods</th>
<th>Falling</th>
<th>Walking/Running</th>
<th>Sitting</th>
<th>Slammed door</th>
<th>Fallen book</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold-based</td>
<td>73 / 0</td>
<td>2240 / 80</td>
<td>48 / 9</td>
<td>19 / 2</td>
<td>62 / 11</td>
</tr>
<tr>
<td>DFT</td>
<td>60 / 13</td>
<td>2320 / 0</td>
<td>57 / 0</td>
<td>19 / 2</td>
<td>73 / 0</td>
</tr>
<tr>
<td>MFCC</td>
<td>66 / 7</td>
<td>2259 / 61</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>69 / 4</td>
</tr>
<tr>
<td>DWT (Haar)</td>
<td>70 / 3</td>
<td>2318 / 2</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>73 / 0</td>
</tr>
<tr>
<td>DWT (Daubechies-2)</td>
<td>71 / 2</td>
<td>2315 / 5</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>73 / 0</td>
</tr>
<tr>
<td>DWT (Daubechies-4)</td>
<td>68 / 5</td>
<td>2318 / 2</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>73 / 0</td>
</tr>
<tr>
<td>DWT (Biorthogonal-3.3)</td>
<td>73 / 0</td>
<td>2315 / 15</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>72 / 1</td>
</tr>
<tr>
<td>DT-CWT</td>
<td>71 / 2</td>
<td>2313 / 7</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>73 / 0</td>
</tr>
<tr>
<td>ST-CWT</td>
<td>72 / 1</td>
<td>2316 / 4</td>
<td>57 / 0</td>
<td>21 / 0</td>
<td>72 / 1</td>
</tr>
</tbody>
</table>
Experimental Results

Change amounts of the energy of the fourth level wavelet coefficients while shifting the 1024-sample-long signal windows.

<table>
<thead>
<tr>
<th></th>
<th>10-sample shifting</th>
<th>15-sample shifting</th>
<th>25-sample shifting</th>
<th>50-sample shifting</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT (Haar)</td>
<td>17.97 %</td>
<td>11.96 %</td>
<td>15.73 %</td>
<td>16.34 %</td>
</tr>
<tr>
<td>DWT (Daubechies-2)</td>
<td>15.44 %</td>
<td>9.37 %</td>
<td>12.72 %</td>
<td>13.59 %</td>
</tr>
<tr>
<td>DWT (Daubechies-4)</td>
<td>12.48 %</td>
<td>6.09 %</td>
<td>10.49 %</td>
<td>10.37 %</td>
</tr>
<tr>
<td>DWT (Biorthogonal-3.3)</td>
<td>20.99 %</td>
<td>15.13 %</td>
<td>15.99 %</td>
<td>21.22 %</td>
</tr>
<tr>
<td>DT-CWT</td>
<td>0.56 %</td>
<td>0.53 %</td>
<td>0.41 %</td>
<td>0.52 %</td>
</tr>
<tr>
<td>ST-CWT</td>
<td>4.32 %</td>
<td>4.48 %</td>
<td>4.57 %</td>
<td>4.34 %</td>
</tr>
</tbody>
</table>
Fall Detection Using Vibration Sensor

• Falling person can be detected only in the falling instant using the vibration sensor.
• There is no chance to analyze the falling event if a sufficient vibration is not sensed on the floor (miss-detection).
• Some of the slow fallings may not be detected using vibration sensor (miss-detection).
• Vibration sensor based system can not sense the activities after falling (e.g. standing up).
Fall Detection Using Two PIR Sensors

- Upper PIR sensor is aligned with the waist
- Upper PIR sensor is turned upside down
- Lower PIR sensor is aligned with the knees
- chipKIT Uno32
Fall Detection Using Two PIR Sensors

- Upper PIR sensor is turned upside down
- Upper PIR sensor is aligned with the waist
- Lower PIR sensor is aligned with the knees
- No motion for a while!
Fall Detection Using Two PIR Sensors
Fall Detection Using Two PIR Sensors
Fall Detection Using Two PIR Sensors

• **Advantages:**
  – Slow fallings can be detected.
  – Sensor signals can be analyzed before and after of the falling event.
  – The system does not affected by the person or the environment.

• **Disadvantage:**
  – This system can not detect instant falling events which are ended up with a fainting.
Multi-Sensor Based Fall Detector

Flowchart:
1. START
2. Read two PIR sensors’ data
   - Motion detection for the lower PIR sensor
     - Store the result
   - Motion detection for the upper PIR sensor
     - Store the result
   - Vibration sensor based human footstep detection and falling person detection systems
3. Unusual inactivity detection system
   - NO: Motion?
   - YES: Analyze the motion activities for last one minute and combine the results of two different falling person detection systems
4. Falling person?
   - NO
   - YES: Decide there is a falling person
Experimental Results

• Vibration and two PIR sensor signals were recorded simultaneously.
• In the some parts of the dataset, one-minute-long signal records contain one falling event.
• Other parts of the dataset contain ordinary activities such as walking, sitting, bending, etc.
• A total of 60 records of one-minute-long signal in the dataset (20 min. falling, 40 min. normal). This is also "falling dataset-2".
### Vibration Sensor Based System

<table>
<thead>
<tr>
<th>Actual Class</th>
<th>Predicted Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Sources</td>
</tr>
<tr>
<td>Other Sources</td>
<td>36</td>
</tr>
<tr>
<td>Falling Person</td>
<td>7</td>
</tr>
</tbody>
</table>

### Two-PIR-Sensor Based System

<table>
<thead>
<tr>
<th>Actual Class</th>
<th>Predicted Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Sources</td>
</tr>
<tr>
<td>Other Sources</td>
<td>38</td>
</tr>
<tr>
<td>Falling Person</td>
<td>5</td>
</tr>
</tbody>
</table>

### Multi-Sensor Based System

<table>
<thead>
<tr>
<th>Actual Class</th>
<th>Predicted Class</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Other Sources</td>
</tr>
<tr>
<td>Other Sources</td>
<td>38</td>
</tr>
<tr>
<td>Falling Person</td>
<td>0</td>
</tr>
</tbody>
</table>

sitting on the floor instantly
slow fallings
sitting on the floor for a while
instant fallings
sitting on the floor for a while
Indoor Flooding Detection

- A remarkable variation does not happened for the PIR sensor signal while the water is running from the tap.
- The vibration sensor based system is developed to benefit from the water dropping sourced vibrations on the floor.
- An adaptive-threshold based method is employed.
- A low-cost simple conductivity based circuit is a better solution to detect indoor flooding.
Stand-Alone AAL System

Uno32 processor connected to the vibration sensor

Uno32 processor connected to two PIR sensors
Dataflow between the Uno32 Boards
An Example Smart Home
Part – V : Conclusion and Future Work

• What we did?

• What is our contribution?

• What will we do?
Conclusion

- Ambient Assisted Living (AAL) system is developed for the purpose of achievement a saleable product.
- The resulting AAL system is a low-cost and privacy-friendly system.
- Vibration and PIR sensor signals are analyzed and some theoretical methods are studied on these signals.
- All detection algorithms are implemented using embedded microprocessors.
Contribution

• Design of a real single-tree lifting-based wavelet transform that possesses complex wavelet-like characteristics.

• Human footstep detection is achieved by using the adaptive-threshold based Markov model classifier.

• Three separate falling person detection algorithms are introduced.

• The stand-alone AAL system using the vibration and two PIR sensors is developed.
Future Works - 1

• Sensitive human footstep detection using the vibration sensor
• Various adaptive systems for the vibration sensor
• Larger dataset, different human activities and environments
• Power consumption calculations for the overall AAL system
• Converting the existing hard-wired networking system to a wireless one
Future Works - 2

• Solution for the false alarm sources such as sitting on the floor for a while
• Generalization of the falling person detection system to handle with different disabilities (paralyzed people, having wheelchair, etc.).
• Analyzing the motion activities of sleeping people to sense a possible problem earlier
• Integration of the gas leak detection and pet detection systems to the overall AAL system
Thank you for listening...

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Extra Slides
Vibration Sensor – AFE Circuit

- Vibrations from distances up to 25 meters
Boundaries of the Sub-Band Frequencies

\[
\left[ 0, \frac{f_S}{256} \right], \left[ \frac{f_S}{256}, \frac{f_S}{128} \right], \left[ \frac{f_S}{128}, \frac{f_S}{64} \right], \left[ \frac{f_S}{64}, \frac{f_S}{32} \right], \left[ \frac{f_S}{32}, \frac{f_S}{16} \right], \left[ \frac{f_S}{16}, \frac{f_S}{8} \right], \left[ \frac{f_S}{8}, \frac{f_S}{4} \right], \left[ \frac{f_S}{4}, \frac{f_S}{2} \right]
\]

if sampling frequency \( f_S = 500 \) Hz

\[
[0, 1.95], [1.95, 3.91], [3.91, 7.81], [7.81, 15.63], [15.63, 31.25], [31.25, 62.5], [62.5, 125], [125, 250]Hz
\]

\[
\left[ 0, \frac{\pi}{128} \right], \left[ \frac{\pi}{128}, \frac{\pi}{64} \right], \left[ \frac{\pi}{64}, \frac{\pi}{32} \right], \left[ \frac{\pi}{32}, \frac{\pi}{16} \right], \left[ \frac{\pi}{16}, \frac{\pi}{8} \right], \left[ \frac{\pi}{8}, \frac{\pi}{4} \right], \left[ \frac{\pi}{4}, \frac{\pi}{2} \right], \left[ \frac{\pi}{2}, \pi \right]
\]

in angular frequency
• **Oscillations:** Since wavelets are band-pass functions, the wavelet coefficients tend to oscillate positive and negative around singularities. This considerably complicates wavelet-based processing, making singularity extraction and signal modeling, in particular, very challenging [6].

• **Shift-variance:** A small shift of the signal greatly perturbs the wavelet coefficient oscillation pattern around singularities. Shift-variance also complicates wavelet-domain processing; algorithms must be made capable of coping with the wide range of possible wavelet coefficient patterns caused by shifted singularities [6].

• **Aliasing:** The wide spacing of the wavelet coefficient samples, or equivalently, the fact that the wavelet coefficients are computed via iterated discrete-time downsampling operations interspersed with nonideal low-pass and high-pass filters, results in substantial aliasing [6].

• **Lack of directionality:** While Fourier sinusoids in higher dimensions correspond to highly directional plane waves, the standard tensor product construction of M-D wavelets produces a checkerboard pattern that is simultaneously oriented along several directions [6].

Analyticity

• A function $f_a \in L^2(R)$ is said to be **analytic** if its Fourier transform is zero for negative frequencies:

\[
\hat{f}_a(w) = 0 \text{ if } w < 0
\]

• An analytic function is necessarily complex but is entirely characterized by its real part.

\[ \psi_c(t) = \psi_h(t) + j\psi_g(t) \]

where \( \psi_h(t) \) and \( \psi_g(t) \) denote the wavelet functions of real and imaginary trees, respectively.

- If \( \psi_c(t) \) is approximately analytic, the resulting transform can possess shift-invariance and lack of aliasing properties just like the Fourier transform.
For $\psi_c(t)$ to be approximately analytic, it is required that one wavelet basis is the approximate Hilbert transform of the other wavelet basis:

$$\psi_g(t) \approx H\{\psi_h(t)\}$$

where $\psi_h(t)$ and $\psi_g(t)$ denote the wavelet functions of real and imaginary trees, respectively.
In order to satisfy $\psi_g(t) \approx H\{\psi_h(t)\}$ condition, low-pass analysis filters in real and imaginary trees must be offset approximately by half-sample [8].

$$g_0[n] \approx h_0[n - 0.5]$$

where $h_0[n]$ and $g_0[n]$ are q-shift filters.

The first stage of the dual-tree filter banks should be different from the other stages. The half-sample delay condition shouldn’t be used for the first stage. For the first stage, it is necessary only to translate one set of filters by one-sample to the other and any set of perfect reconstruction filter can be used.

The key properties required for the q-shift (quarter sample shift orthogonal) filters are that they should provide a group delay of either 1/4 or 3/4 of a sample period, while also satisfying the usual 2-band filter-bank constraints of no aliasing and perfect reconstruction [7].

• Satisfying linear-phase property of $h_0[n]$ is achieved by

$$g_0[n] = h_0[N - 1 - n]$$

where $N$ (even) is the length of $h_0[n]$.

• In q-shift method the imaginary part of the complex wavelet is a time-reversed of the real part.

Even and odd samples of the input signal can be obtained using a lazy filter-bank.

Half-Band Filters - 1

• An $L$th-band filter with $L = 2$ is called a half-band filter.

\[
H(z) = \alpha + z^{-1}U(z^2)
\]

with its impulse response satisfying

\[
h[2n] = 0 \text{ for } n \neq 0, \text{ and } h[0] = \alpha
\]

• If $\alpha = 0.5$, then $H(z) + H(-z) = 1$. 
• $H(e^{j\omega})$ exhibits a symmetry with respect to the half-band frequency $\frac{\pi}{2}$, hence the name of the filter is "half-band".

• **Attractive property:** About 50% of the coefficients of $h[n]$ are zero. This reduces the number of multiplications required in its implementation significantly.
• The high-pass filter $\tilde{g}_1[n]$ is formed from the low-pass filter $\tilde{h}_1[n]$ as follows:

$$\tilde{g}_1[n] = (-1)^n \tilde{h}_1[N - 1 - n]$$

where $N$ is the length of the $\tilde{h}_1[n]$.

• The second high-pass filter $\tilde{g}_2[n]$ is simply the time-reversed of the high-pass filter $\tilde{g}_1[n]$.