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An Online Adaptive Cooperation Scheme for Spectrum Sensing Based on a Second-order Statistical Method

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***Abstract*—Spectrum sensing is one of the most important features of cognitive radio (CR) systems. Even though spectrum sensing can be performed by a single CR, it is shown in the literature that cooperative techniques including multiple CRs/sensors improve performance and reliability of spectrum sensing. Existing cooperation techniques usually assume a static communication scenario between the unknown source and sensors along with a fixed propagation environment class. In this study, an online adaptive cooperation scheme is proposed for spectrum sensing in order to maintain the level of sensing reliability and performance under changing channel and environmental conditions. Each cooperating sensor analyzes second-order statistics of the received signal which undergoes both correlated fast– and slow–fading. Autocorrelation estimation data from sensors are fused together by an adaptive weighted linear combination at the fusion center. Weight update operation is performed online through the use of orthogonal projections onto convex sets (POCS). Numerical results show that the performance of the proposed scheme is maintained for dynamically changing characteristics of the channel between an unknown source and sensors, even under different physical propagation environments. Also it is shown that the proposed cooperative scheme which is based on second–order detectors yields better results compared to the same fusion mechanism that is based on conventional energy detectors.**

***Index Terms*—adaptive data fusion, online learning, fast fading, mobility, shadowing, spectrum sensing, POCS**

I. Introduction

Cognitive radio (CR) systems, which are aware of their surroundings and have the capability of self-adaptation to dynamic environmental and channel conditions, have emerged as a novel paradigm in wireless communications [1]. One of the most distinguished features of these systems is spectrum sensing for dynamic spectrum access. Dynamic spectrum access consists mainly of the following steps: (i) observing a specific portion of the radio frequency (RF) spectrum steadily, (ii) deciding whether the portion of interest is occupied or not, and (iii) exploiting the opportunities in such a way that no harm is done to primary users. From the perspective of both (ii) and (iii), it can be said that agility and accuracy are the two prominent requirements for CRs in spectrum sensing, since CRs need to be accurate in their decisions about whether there is a spectrum opportunity. Furthermore, once an opportunity (or a licensed user) emerges they need to be very agile to take appropriate actions (*e.g.*, exploiting a white hole or vacating the band due to an emerging primary user) in a timely manner.

Accuracy in spectrum sensing is generally inversely proportional to the complexity of CR systems. However, an increase in complexity implies a decrease in agility, which constitutes a critical design trade–off [2, 3]. Therefore, most of the times it is difficult to attain both the utmost agility and high accuracy in receiver design simultaneously. Especially in spectrum sensing, such trade–offs lead to various methods described in the literature which range from those which give priority to agility to those which give priority to accuracy. As will be discussed subsequently, all of these methods are somehow related to the amount of knowledge in hand before spectrum sensing operations take place.

Spectrum sensing can generally be considered under the absence or presence of *a priori* knowledge of the signal to be detected. In case there is no knowledge about the signal, it is shown that the optimal detector is "energy detector," which is a non–coherent receiver measuring solely the energy of the received signal over a specific period of time for a specific band [4]. The energy detector has very critical drawbacks such as being prone to uncertainties in noise variance [5], exhibiting degraded performance for low signal–to–noise ratio (SNR) values [6], and performing unsatisfactorily, especially in detecting spread spectrum signals [7]. As opposed to no–knowledge case, when the signal itself is completely known, the optimal detection in a stationary Gaussian noise is a matched filter (with a threshold comparison) [8]. Note that having a complete knowledge of a signal includes a very long and detailed set of parameters some of which are signaling bandwidth, operating frequency, modulation type and order, pulse shape, frame/burst format and so on. As can be seen from this list, the optimal solution under certain assumptions comes at the expense of obtaining a very broad knowledge of the signal. It is obvious that such a broad knowledge might not always be available. Instead of having a complete knowledge, partial knowledge of the signal might be available, such as the signal to be detected being digitally modulated

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with a certain (known) symbol rate. In such scenarios, more complex architectures, which still include matched filters, can safely be used in an optimal sense [9]. Another approach in partial knowledge scenarios is known as "waveform–based sensing." In waveform–based sensing, a set of known characteristics of the signal whose absence/presence is to be detected is searched across the received signal with the use of specially constructed templates [6, 10]. The main drawback of waveform–based sensing methods is that they still need to rely on the knowledge of some distinct characteristics of the transmitted signal even though the knowledge is partial. Slightly different from waveform–based sensing approach, cyclostationarity–based methods strive to exploit the inherent periodicity in the statistics of a signal such as its autocorrelation [11, 12]. This way, instead of focusing directly on the knowledge of the signal itself, an indirect approach is adopted by investigating its inherent statistical features. Especially under the "signal interception" umbrella, more comprehensive and unifying studies based on several inherent statistical characteristics of very broad classes of signals can also be found in the literature [13]. Despite the fact that it is a very powerful method, cyclostationarity–based methods might be relatively more computationally complex. Similar to cyclostationarity–based methods, correlation–based approaches can also be applied to spectrum sensing problems [14–16]. However, spatial correlation of shadowing needs to be taken into account, since it changes the statistical characteristics of signals and affects the performance [16]. Apart from these, there are also some studies that consider multi–level sensing. For instance, in radio identification based sensing, some approaches extract features of the signal first and then these features are fed to another level to identify the absence/presence of a primary user. Another approach in multi–level sensing is to take into account both local and global decisions along with some sort of decision rule. For a very comprehensive list of studies including various other methods on spectrum sensing available in the literature, the readers might refer to [17–22, and references therein].

Up until this point, spectrum sensing methods are reviewed from the perspective of a single CR or sensor.[1] However, there are scenarios in which multiple sensors might be used in spectrum sensing. When multiple sensors are involved in spectrum sensing, several concerns arise: (C.i) First of all, many sensors imply many input data; therefore, a decision/data fusion is essential in order to come up with a single decision on the absence/presence of an unknown source. Decision fusion forces one to contemplate a fusion architecture. (C.ii) Second, once a plausible architecture is proposed, a mechanism needs to be established so that the deficiency in observations of each individual sensor stemming from receiver uncertainty, fading, and shadowing will be overcome. Hence, cooperative spectrum sensing schemes are proposed by taking into account both (C.i) and (C.ii) to improve the sensing performance through the use of spatial diversity [19, 23–26]. Even though there is vast and ever-increasing studies in the literature on cooperative spectrum sensing [19, 26, and references therein], most of the existing methods are developed based on a static communication scenario between the unknown source and the sensors along with fixed RF propagation environment classes and characteristics. However, it would be too optimistic to state that these cooperative techniques, with their non-adaptive nature, can survive in practice under dynamically shifting channel and environmental conditions.

[1]From this point on the terms "CR" and "sensor" are used interchangeably.

Online learning approaches are powerful tools for problems where drifts in concepts take place. It is important to observe that many CR problems accommodate dynamic characteristics in many aspects. These dynamic characteristics cause drifts in paradigms. For instance, mobility not only causes correlated fast–fading, but also leads to changes in propagation environment class or characteristics such as moving from urban to suburban area. Since such drastic changes need to be taken into account in spectrum sensing, an online learning approach seems very promising especially from the perspective of practical CR systems. Therefore, in this study, an adaptive data fusion (ADF) scheme, which exploits inherent dynamics of the sensing problem by adapting the weight of the contribution from each spectrum sensor in an online manner, is proposed for cooperative spectrum sensing. Each sensor carries out a set of operations based on second–order statistical characteristics of the received unknown signal, which is assumed to be emitted by a mobile source. Considering the fact that shadowing process changes slowly compared to fast–fading process, detectors aim to separate the statistics of these processes from the received signal by employing a low–pass filter followed by a logarithmic detector and investigating the second–order statistics of the output. Then, the output value of each sensor is sent to the fusion center for their corresponding weights to be updated online based on the ADF scheme through the use of orthogonal projections onto convex sets (POCS). In this regard, the contributions of this study can be summarized as follows: (C.I) All of the main propagation mechanisms in the physical layer such as correlated fast– and slow–fading are taken into account in each spectrum sensor; (C.II) a second–order detector is employed in sensors which can decide individually on the absence/presence of the unknown signal; (C.III) an adaptive cooperation scheme is proposed to maintain the reliability of the spectrum sensing system online by tracking the dynamics in the channel and the propagation environment via the output of the second–order detectors. Note that, the general framework of a POCS based online adaptive decision fusion scheme was introduced in [27] for a machine vision application. This general framework was then applied for a cooperative spectrum sensing problem where each sensor utilizes an energy-based spectrum sensing approach in [28, 29]. Different from the previous studies, in this paper, the fusion mechanism exploits bounded input generated by second-order detectors.

The organization of the paper is as follows. The system model and the statement of the problem are presented in Section II. In Section III, the proposed online adaptation and data fusion method is described. Numerical results and discussions are presented in Section IV. Finally, conclusions are drawn in the last section.

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II. System Model and Statement of the Problem

Let a fixed, immobile spectrum sensor network be composed of M sensors, possibly situated at M geographically different locations. Each sensor, say Sensor$_i$, where $i = 1, \ldots, M$, carries out a sequence of operations upon receiving a signal $r_i(\cdot)$ coming from a single unknown radio source. Next, each sensor yields an output value $V_U^{(i)}[n]$ at a discrete time index, say n, and sends it over a broadcast channel to a fusion center. In the final step, fusion center takes all discrete input collected over broadcast channel, say $\{y_i[n]\}$, and combines them in such a way that at the end a binary decision is performed based on a threshold under certain conditions such as a fixed probability of false alarm. It is worth mentioning here that spectrum assignment is assumed to be already established before the system runs and maintained throughout the entire period of operation. Broadcast fusion channel is assumed to function of simplex mode, which solely conveys sensor output to the fusion center. In case a new spectrum assignment is required, then the new assignment information is assumed to be dispatched to the sensors through the use of a reserved control channel. An outline of the system model described here is depicted in Figure 1.

In light of the system model given above, the problem can be stated as follows: identifying the absence/presence of an unknown narrow band radio source by analyzing it with M different, independent, and individual sensors and then coming up with a binary decision on the absence/presence of the unknown source by fusing output of the broadcast channel carrying sensor output values under a specific probability of false alarm value.

Stemming from the fact that the system model considered in this study functions in multiple steps as discussed above, it is useful to contemplate each step individually. However, in order to formalize the overall behavior of the system model, let the following be given in regards to input and output of the entire system considered in here:

$$r_i(t) = \begin{cases} \qquad\quad n_i(t), & H_0, \\ x_i(t) \;+\; n_i(t), & H_1, \end{cases} \tag{1}$$

where $x_i(t)$ is the output of the channel (including slow- and fast-fading) between the unknown source and Sensor$_i$; $n_i(t)$ is the ambient noise at the antenna of the Sensor$_i$. In (1), H_0 corresponds to the case where the unknown source is absent, whereas H_1 corresponds to the case where the unknown source is present. It is important to keep in mind that the absence/presence of the unknown source is universal for each and every Sensor$_i$ since the system model described above assumes that there is a single unknown radio source. However, the system model given above is not limited to identifying the absence/presence of a single unknown source. In order to see this one can imagine that there is more than one unknown active source. It is clear that there is no difference between a single active unknown source and multiple active unknown sources since there is at least one unknown activity in either case. On the contrary, the absence of unknown source requires that there is absolutely no active unknown source. Since the difference between absence and presence of unknown source is solely a single active unknown source, this study focuses on a single unknown source scenario.

A. Channel Characteristics Between Sensors and The Unknown Source

By adopting complex baseband representation, for each Sensor$_i$, noise process $n_i(t)$ is assumed to be of complex additive white Gaussian noise (AWGN) form with $\mathcal{CN}\left(0, \sigma_N^2\right)$ as $n_i(t) = n_i^I(t) + j n_i^Q(t)$ where both $n_i^I(t)$ and $n_i^Q(t)$ are $\mathcal{N}\left(0, \sigma_N^2/2\right)$ and $j = \sqrt{-1}$. On the other hand, $\{x_i(t)\}$ are assumed to be narrowband signals where the delay spread of each channel (the channel between the unknown source and Sensor$_i$) is relatively small compared to the inverse bandwidth of the channel of interest. Therefore, under the narrowband assumption for Sensor$_i$, the unknown signal $x_i(t)$ can be modeled by decomposing it into the following form:

$$x_i(t) = m_i(t) s_i(t) a(t), \tag{2}$$

where $m_i(t) = h_i(t) e^{j\theta_i(t)}$ represents the complex fading channel process whose amplitude and phase are denoted with $h_i(t)$ and $\theta_i(t)$, respectively; $s_i(t)$ denotes the real–valued slow–fading process including the combined effects of both distance–dependent path loss and shadowing; and $a(t)$ is the unknown baseband signal. In addition, all three processes in (2) are assumed to be independent of each other and of the noise process $n_i(t)$.

Note that for the sake of notational convenience, the index i representing i–th sensor will be dropped and all analyses will hereafter be carried out for a single, generic sensor, unless otherwise stated.

Complex fading channel process $m(t)$ is composed of multiple rays (sometimes referred to as paths) arriving at the receiver antenna and causes rapid fluctuations in the power level of the received signal with respect to very small displacements on the order of a couple of wavelengths of the transmission. Complex fading channel process is mainly characterized by the distribution of its fading amplitude $h(t) = |m(t)|$. In the literature, some of the frequently used fading amplitude distributions are Rayleigh, Rice, and Nakagami–m distributions. Beside amplitude distribution, Doppler spectrum of the fading channel process is also important in characterizing the complex fading process [30]. Two of the frequently used models in the literature for Doppler spectrum are Jakes' and Gaussian Doppler spectrum.

It is known that both path loss and shadowing change more slowly compared to the fast fading process $m(t)$. Therefore, there is no harm in modeling both path loss and shadowing with a single process as follows [31, 32]:

$$s(t) = \exp\left(\frac{1}{2}\mu(t) + \frac{\sigma_G}{2} g(t)\right), \tag{3}$$

where $\mu(t)/2$ denotes mean, $\sigma_G/2$ is the standard deviation of log–normal shadowing, and $g(t)$ is a real–valued unit normal process $\mathcal{N}(0,1)$. Moreover, the experimental studies present in the literature for shadowing process $g(t)$ show that shadowing correlation can be approximated by the following model [33]:

$$R_g(\tau) = E\left\{g(t)g(t+\tau)\right\} = \exp\left(-\frac{v\,|\tau|}{d_\rho}\right) \tag{4}$$

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Note also that there are some other studies in the literature related to shadowing models, such as static and dynamic shadowing [34]. Yet, the model defined by both (3) and (4) are adopted due to the following two reasons: (R.1) It is clear that due to the mathematical tractability of both (3) and (4), the analysis will be simpler. Furthermore, such a sharp (exponential) decay yields pessimistic results in terms of shadowing correlation, which provide some sort of upper bound for the problem considered. Having said this, as will be shown subsequently, it is important to mention that the proposed method is independent of any sort of shadowing correlation model.

Finally, without loss of generality, it is assumed that displacement of the unknown source within the duration of operation is negligibly small compared to the distance between the unknown signal source and the sensors. Therefore, the impact of $\mu(t)$ can be neglected so that $s(t)$ is assumed to solely include the impact of the shadowing process.

B. Channel Characteristics Between Fusion Center and Sensors

The channel between the fusion center and the sensors is called "broadcast fusion channel." As opposed to the channel between sensors and an unknown source, the broadcast fusion channel is considered to be of discrete form for the sake of notational convenience. However, this assumption does not affect the essence of the method proposed at the fusion center.

Since both sensors and the fusion center (and even the objects in between) are assumed to be immobile, one can assume that the broadcast channel gains do not change in time. Because there is ambient noise at the fusion center, the broadcast fusion channel can be considered to be an AWGN channel. Note that such conditions are valid for transmissions through a guided media, such as a direct cable connection between the fusion center and the sensors. Also, fixed scenarios such as rooftop–to–rooftop communications with the presence of a very strong line–of–sight (LOS) can be considered to be AWGN channel [35, 36]. Such scenarios are nothing but special versions of a more generalized scheme where there is a stationary transmitter and a fixed receiver that is equipped with an antenna of high directivity. In such generalized schemes, the channel between the transmitter and the receiver falls into AWGN channel category because directivity of the receiver antenna can be adjusted in such a way that either direction of the strong LOS or the strongest path is aimed at [37, 38]. Furthermore, it is known that under fairly general conditions, fading channels can be transformed into AWGN form by increasing the number of diversity branches [39]. In this regard, the set of signals reaches the fusion center at n–th discrete time instant can be modeled as a Gaussian channel with zero–mean noise q_i and with variance $\delta = [\delta_1^2, \delta_2^2, \ldots, \delta_M^2]^T$ for the sake of an easier analysis:

$$\mathbf{y} = \mathbf{V_U} + \mathbf{q} \tag{5}$$

with $\mathbf{y} = [y_1[n], \ldots, y_M[n]]^T$, $\mathbf{V_U} = \left[V_U^{(1)}[n], \ldots, V_U^{(M)}[n]\right]^T$ and $\mathbf{q} = [q_1[n], \ldots, q_M[n]]^T$ where $(\cdot)^T$ denotes the transpose operation.

III. Proposed Method

In this section, the details of how sensors operate to come up with their output values, namely $V_U^{(i)}[n]$, will be discussed. Following that, the analysis of the fusion center will be investigated.

A. Sensors

In Section II, it is stated that statistics of the shadowing and fast–fading processes evolve in different scales on spatial domain. This implies that the shadowing process is not expected to vary within relatively short displacements, such as in a couple of wavelengths of the transmission. Keeping this in mind, first consider passing the received signal through a low–pass filter whose (normalized) impulse response is given by:

$$w(t) = \frac{1}{2T_A}\left(\mathrm{sgn}\left(t + \frac{T_A}{2}\right) - \mathrm{sgn}\left(t - \frac{T_A}{2}\right)\right) \tag{6}$$

where $\mathrm{sgn}(\cdot)$ is the signum (or sign) function and T_A denotes the effective averaging duration. Now, consider the hypothesis H_1, since it includes both noise and the unknown signal terms. If $r(t)$ is passed through the low–pass filter under the hypothesis H_1, then, in light of both (2) and (6) and after some mathematical manipulations [16]:

$$\begin{aligned} z(t) &= s(t) \underbrace{\int_{t-T_A/2}^{t+T_A/2} w(t-\tau)m(\tau)a(\tau)\, d\tau}_{M_F(t)} + N_F(t) \\ &= s(t)M_F(t) + N_F(t). \end{aligned} \tag{7}$$

is obtained where $N_F(t)$ denotes the low–pass filtered white Gaussian noise (WGN) and T_A is assumed to be so short that shadowing does not change within.[2]

Next, the natural logarithm operator is applied to the absolute square of $z(t)$ in order to reveal the impact of the shadowing process, which reads:

$$\ln(Z(t)) = \ln\left(s^2(t)\,|M_F(t)|^2\right) + \ln\left(\frac{Z(t)}{|s(t)M_F(t)|^2}\right). \tag{8}$$

With the aid of both (3) and (8):

$$\ln(Z(t)) = \sigma_G g(t) + \underbrace{\ln\left(|M_F(t)|^2\right) + \ln\left(\frac{Z(t)}{|s(t)M_F(t)|^2}\right)}_{L(t)}. \tag{9}$$

is obtained by neglecting the impact of distance–dependent path loss.

It is clear that the autocorrelation of (9) will include the shadowing correlation via $g(t)$. Therefore, the unbiased estimate of autocorrelation of (9) is found to be:

$$R_{\ln(Z)}(\tau) = \sigma_G^2 e^{-v|\tau|/d_\rho} + R_L(\tau) + R_{gL}(\tau) + R_{Lg}(\tau). \tag{10}$$

[2]A brief discussion regarding to what extent T_A can be considered to be short is given in Section IV for both practical scenarios and general cases.

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Although the impact of shadowing is clear in (10), it is difficult to come up with a definitive statement in regards to the presence of an unknown signal. This mainly stems from the following two reasons: First, remnants of low–pass filtering operation cannot be removed completely. Second, the autocorrelation estimates are biased with the mean of $\ln(Z(t))$. In addition to these two, one should also keep in mind that finite support leads to drastic fluctuations in autocorrelation estimates and renders the entire decision process difficult. However, as will be shown subsequently, all these issues can be remedied by investigating the noise–only process through the steps (7)–(10). In the following parts, (10) will be discussed further in light of the observations for the noise–only case. First, the following needs to be given.

Proposition 1. *Under the hypothesis H_0 along with the ideal conditions such as $T \rightarrow \infty$, $T_A = 0$, and unit variance noise, normalized output of the correlator of any Sensor$_i$ converges the constant $\Phi' = \frac{\gamma^2}{\gamma^2+\pi^2/6}$ where γ is Euler–Mascheroni (or sometimes referred to solely as Euler's) constant.*

Proof: See Appendix A. ∎

Note that $T_A = 0$ implies no low–pass filtering operation. It is clear that a low–pass filter applied prior to the logarithmic detector will not alter the convergent behavior of Φ'. But it will change the value of Φ' to a new constant, say Φ''_{T_A}. Since there is no closed form, by relaxing some of the conditions imposed in Appendix A, Φ''_{T_A} can be approximated with $\Phi''_{T_A} = \frac{k^2\gamma^2}{k^2\gamma^2+\pi^2/6}$ as an extension to (42). Here, $k \in \mathbb{Z}^+$ denotes the number of samples which are taken $1/f_s$ seconds apart and f_s is the sampling frequency of the receiver.

Since finite support leads to fluctuations around Φ''_{T_A} at larger τ, the autocorrelation estimates in (10) are fed into the following unbiased estimator prior to the decision step:

$$V_U = \frac{1}{U} \int_{T_A}^{T_A+U} R_{\ln(Z)}(\tau) d\tau \tag{11}$$

where U denotes the effective integration time. It is clear that when $T_A = 0^+$ and $U \rightarrow \infty$, then $V_U \rightarrow \Phi'$.

Analysis of noise–only case reveals that a drastic drop is anticipated at the output of the correlator for H_0. On the other hand, for $x(t) \neq 0$, even though a drop will still be observed, it will not be as drastic as that in the noise–only case.[3] Therefore, one can conclude that:

$$\Phi''_{T_A} < V_U \tag{12}$$

always holds. Moreover, as shown in Appendix A, Φ''_{T_A} implies that no such measurement is required to determine a specific threshold.

Considering the practical aspects, T_A can be selected to be the lowest non–zero value that is possible at the receiver. This way, the assumption regarding the invariance of shadowing is still maintained. On the contrary, U should be chosen as large as possible to obtain better estimates of V_U in (11). Therefore, one can conclude that U is actually bounded by the memory (or buffer) capacity of the receiver.

In the sequel, it is critical to emphasize that each and every sensor is actually able to make a decision on the absence/presence of an unknown source through the use of V_U as explained in [16]. However, in this study, the scenario in which decisions are made by individual sensors are not considered. Therefore, it is assumed that each Sensor$_i$ sends the output of its correlator at time instant n, namely $V_U^{(i)}[n]$, to the fusion center through the use of a broadcast channel.

[3]This is very critical because of the reason (R2) stated in Section II. Considering the fact that the proposed method is independent of any specific correlation model for shadowing, Φ''_{T_A} constitutes the lower bound for the problem considered here.

B. Fusion Center

Sensors process received signals in accordance with the second–order statistical method whose details are given in Section III-A. Here, it is important to recall that the output of each sensor can be considered to be sent at discrete time instants. Sensors output are transmitted over a very narrow–band channel, namely the broadcast fusion channel, to an immobile "fusion center." In this regard, based also on the practical layout scenarios introduced in Section II-B such as rooftop–to–rooftop (or through the use of a guided media) communications, the set of signals reaches at the fusion center at n–th discrete time instant is assumed to be modeled as expressed in (5). At the fusion center, where the adaptive data fusion is realized online, a decision is made through the use of a global test statistic $y_c[n]$ that is computed from $y_i[n]$ as follows:

$$y_c[n] \gtrless_{\mathcal{H}_0}^{\mathcal{H}_1} \gamma_c[n] \tag{13}$$

where

$$\begin{aligned} y_c[n] &= \sum_{i=1}^{M} y_i[n]\, w_i[n] \\ &= \mathbf{y}^T[n]\,\mathbf{w}[n] \end{aligned} \tag{14}$$

and

$$\mathbf{w}[n] = [w_1[n], \ldots, w_M[n]]^T, \qquad w_i[n] \geq 0 \tag{15}$$

Weight vector $\mathbf{w}[n]$ corresponding to spectrum sensors is updated in order to maintain the same P_f under dynamically changing channel and propagation environment characteristics. Physically, weight vectors are affected by mobility, shadowing and type of the propagation environment, such as urban or suburban. Dynamical changes in the environment and channel directly affects the output of the sensors, V_Us, which consequently affect the weights at the fusion center.

Recall from Section III-A that under hypothesis H_0, output of the correlator converges to a constant ($V_U \rightarrow \Phi'$), whereas it yields always greater values, say Φ^+, than the constant Φ' for the same settings ($T_A = 0$) under the hypothesis H_1. As will also be shown in Section IV, output of correlators under the hypothesis H_1 can be assumed to be constant as well with the aid of the unbiased estimator given in (11). It is a key observation that the constant Φ' is universal for all sorts of environmental classes such as urban and suburban, whereas Φ^+ changes from one environment to another. Although Φ^+ changes depending on the environment, within the decision process assuming that environmental class does not change,

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Φ^+ is considered to be a constant. Therefore, it is clear that $\mathbf{y}$ in (5) is actually another normal random variable because both Φ' and Φ^+ are constants. Bearing in mind that the linear combination of normal random variables yields another normal random variable (with probably different mean and standard deviation), the expected value of the weighted combination for the unit variance noise at the sensor input at any time instant n, namely $y_c[n]$, is given by:

$$E\{y_c[n]\} = \begin{cases} \Phi'\|\mathbf{w}[n]\|_{\mathbf{1}}, & \text{if } H_0 \\ \Phi^+\|\mathbf{w}[n]\|_{\mathbf{1}}, & \text{if } H_1 \end{cases} \tag{16}$$

where $\Phi' < \Phi^+$ and $\|\cdot\|_{\mathbf{1}}$ is the $\mathbf{L_1}$–norm since $\{w_i\}$ are defined to be non–negative in (15). Variance of $y_c[n]$ can then be calculated via (16) as:

$$\begin{aligned} E\left\{(y_c[n] - E\{y_c[n]\})^2\right\} &= \mathbf{w}^T[n]\mathrm{Cov}_{\mathbf{q[n]}}\mathbf{w}[n] \\ &= \sum_{i=1}^{M}(w_i\delta_i)^2 \end{aligned} \tag{17}$$

for both H_0 and H_1 where $\mathrm{Cov}_{\mathbf{q[n]}}$ denotes the covariance matrix of $\mathbf{q}[n]$. The key observation here is that output of sensors are actually constants whose values are dependent on the hypothesis H_0 or H_1. According to the system model given in Section II, this implies that the fusion center combines linearly output of correlators (*i.e.*, constants) disturbed by AWGN. Since the noise in broadcast fusion channel is assumed to be of AWGN form, the linear combination in the fusion center yields another Gaussian random variable whose mean and variance are given in (16) and (17). Hence, the performance metric used for the system model is the following $\{P_f, P_d\}$ pair due to the aforementioned Gaussian assumption:

$$P_f = Q\left[\frac{\gamma_c[n] - \Phi'\|\mathbf{w}[n]\|_{\mathbf{1}}}{\sqrt{\mathbf{w}^T[n]\mathrm{Cov}_{\mathbf{q[n]}}\mathbf{w}[n]}}\right] \tag{18}$$

and

$$P_d = Q\left[\frac{\gamma_c[n] - \Phi^+\|\mathbf{w}[n]\|_{\mathbf{1}}}{\sqrt{\mathbf{w^T}[n]\mathrm{Cov}_{\mathbf{q[n]}}\mathbf{w}[n]}}\right] \tag{19}$$

where P_f denotes the probability of false alarm and P_d denotes the probability of detection, and $Q(\cdot)$ is the complementary cumulative distribution function, which calculates the tail probability of a zero mean unit variance Gaussian random variable.

Here note that different weight selection rules can be employed for different purposes, such as the one described in [40, 41]. In this study, although the broadcast fusion channel is assumed to be of AWGN form, there is no restriction imposed on the channel (and on the type of propagation environment such as urban or suburban) between the sensors and the unknown signal source. Therefore, the system model can easily be extended to the one in which unknown signal $x(t)$ can be assumed to undergo shadowing, multipath fading, and Doppler spread (i.e., a mobile unknown signal source) as well.

As soon as $y_c[n]$ is calculated, an estimate of the test threshold for the corresponding time step n should be calculated so that an error is obtained and the weights are updated accordingly. As discussed earlier, at each time step n, $\gamma_c[n]$ is calculated with the aid of both (18) and (19). Hence, the error value for the corresponding time step is calculated as:

$$e_c[n] = \gamma_c[n] - y_c[n]. \tag{20}$$

For a fixed value of probability of false alarm, P_f, one can obtain the corresponding threshold value, $\gamma_c[n]$, from (18) as:

$$\gamma_c[n] = \Phi'\|\mathbf{w}[n]\|_{\mathbf{1}} + Q^{-1}(P_f)\sqrt{\mathbf{w^T}[n]\mathrm{Cov}_{\mathbf{q[n]}}\mathbf{w}[n]} \tag{21}$$

Along with the weight update equation to be presented, (21) provides self-adaptation of weights in such a way that statistics of P_f are not affected by dynamic changes and drifts in the channel and/or the propagation environment. Considering the dynamic changes in the physical environment, in order to have a cooperative spectrum sensing system which maintains certain performance criteria while avoiding any assumptions on the physical world such as number of users, type of the propagation medium, etc., one possible way is to incorporate a controlled feedback mechanism based on an error term, $e_c[\cdot]$ to the decision making strategy. In the proposed online ADF framework, this is achieved by keeping the false alarm rate fixed which in turn implies a constant value for the threshold, $\gamma_c[n]$ in (21). Consequently, at each time step, the error value is evaluated as in (20) with respect to test threshold. One of the main advantages of the proposed online cooperative spectrum sensing strategy is this feedback mechanism, as compared to other related methods like those discussed in [24]. The weights of the spectrum sensors yielding correlation estimates different than (same as) the test threshold are reduced (increased) iteratively at each time step, making it possible to keep the performance of sensing unaffected by the change in channel characteristics. It is worth mentioning at this point that the proposed algorithm is independent of any specific probability distribution on the data. Also, as discussed earlier in this section, different values of Φ^+ for different environmental classes do not affect the analysis, since (21) is independent of Φ^+.

Set Theoretic Weight Update Algorithm: Ideally, the weighted sum of the received summary statistics of spectrum sensors should be equal to the test threshold $\gamma_c[n]$ at the time instant n:

$$\gamma_c[n] = \mathbf{y}^T[n]\mathbf{w}[n] \tag{22}$$

which represents a hyperplane in the M–dimensional space, $\mathbf{w}[n] \in \mathbb{R}^M$. Hyperplanes are convex in $\mathbb{R}^M$. At time instant n, $\mathbf{y}^T[n]\mathbf{w}[n]$ may not be equal to $\gamma_c[n]$. The next set of weights are determined by projecting the current weight vector $\mathbf{w}[n]$ onto the hyperplane represented by (22). This process is geometrically depicted in Figure 2. The orthogonal projection $\mathbf{w}[n+1]$ of the vector of weights $\mathbf{w}[n] \in \mathbb{R}^M$ onto the hyperplane $\gamma_c[n] = \mathbf{y}^T[n]\mathbf{w}[n]$ is the closest vector on the hyperplane to the vector $\mathbf{w}[n]$.

Let us formulate the problem as a minimization problem:

$$\min_{\mathbf{w}^*} |\mathbf{w}^* - \mathbf{w}[n]|, \quad \text{subject to: } \mathbf{y}^T[n]\mathbf{w}^* = \gamma_c[n] \tag{23}$$

The solution can be obtained by using Lagrange multipliers:

$$\mathcal{L} = \sum\nolimits_i (w_i[n] - w_i^*)^2 + \lambda\left(\mathbf{y}^T[n]\mathbf{w}^* - \gamma_c[n]\right) \tag{24}$$

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taking partial derivatives with respect to w_i^*:

$$\frac{\partial \mathcal{L}}{\partial w_i^*} = 2(w_i[n] - w_i^*) + \lambda y_i[n], \quad i = 1, \ldots, M, \tag{25}$$

setting the result to zero:

$$2(w_i[n] - w_i^*) + \lambda y_i[n] = 0, \quad i = 1, \ldots, M \tag{26}$$

and defining the next set of weights as $\mathbf{w}[n+1] = \mathbf{w}^*$ a set of M equations is obtained:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{\lambda}{2}\mathbf{y}[n] \tag{27}$$

The Lagrange multiplier, λ, can be obtained from the condition equation:

$$\mathbf{y}^T[n]\,\mathbf{w}^* - \gamma_c[n] = 0 \tag{28}$$

as follows:

$$\lambda = 2\frac{\gamma_c[n] - y_c[n]}{||\mathbf{y}[n]||^2} = 2\frac{e_c[n]}{||\mathbf{y}[n]||^2} \tag{29}$$

where the error term is given by (20). Plugging this into (27):

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \frac{e_c[n]}{||\mathbf{y}[n]||^2}\mathbf{y}[n] \tag{30}$$

is obtained. Hence, the projection vector is calculated according to (30).

Whenever a new set of correlator estimates, $\mathbf{V_U}$, are generated by spectrum sensors, another hyperplane based on the new data values $\mathbf{y}[n]$ arrived at the fusion center from the broadcast fusion channel, is defined in $\mathbb{R}^M$:

$$\gamma_c[n+1] = \mathbf{y}^T[n+1]\,\mathbf{w}^* \tag{31}$$

This hyperplane will probably not be the same as $\gamma_c[n] = \mathbf{y}^T[n]\,\mathbf{w}[n]$ hyperplane as shown in Figure 2. The next set of weights, $\mathbf{w}[n+2]$, are determined by projecting $\mathbf{w}[n+1]$ onto the hyperplane in (31). Iterated weights converge at the intersection of hyperplanes, $\mathbf{w^c}$, as stated in [42]. The rate of convergence can be adjusted by introducing a relaxation parameter μ to (30) as follows:

$$\mathbf{w}[n+1] = \mathbf{w}[n] + \mu\frac{e_c[n]}{||\mathbf{y}[n]||^2}\mathbf{y}[n] \tag{32}$$

where $0 < \mu < 2$ should be satisfied to guarantee the convergence according to the POCS theory [43, 44].

The relaxation parameter has an important role in the convergence speed of POCS based algorithms as this has been very well analyzed in the literature under the assumption of having a wide–sense stationary (WSS) model [45, and references therein]. However, convergence may take infinitely long for the case where the hyperplanes in Figure 2 do not intersect at all. This is true for most of the practical cases and spectrum sensing is not an exception. In case the intersection of hyperplanes is an empty set, the updated weight vector simply satisfies the last hyperplane equation. In other words, it tracks the test threshold, $\gamma_c[n]$, by assigning proper weights to individual spectrum sensors, in order to maintain the same P_f value under dynamically changing channel and propagation environment characteristics. Note that the proposed online decision fusion method does not need to wait for convergence to give a decision.

Algorithm 1 The pseudo-code for the Adaptive Data Fusion (ADF) algorithm

```
Adaptive Data Fusion[n]
for i = 1 to M do
    w_i[0] = 1/M, Initialization
end for
γ_c[n] = Φ'||w[n]||_1 + Q^{-1}(P_f) sqrt( w^T[n] Cov_{q[n]} w[n] )
e_c[n] = γ_c[n] − y_c[n]
for i = 1 to M do
    w_i[n+1] ← w_i[n] + μ (e_c[n] / ||y[n]||^2) y_i[n]
end for
y_c[n] = Σ_i w_i[n] y_i[n]
if y_c[n] ≥ γ_c[n] then
    return H_1
else
    return H_0
end if
```

IV. Numerical Results and Discussions

In simulations, multiple spectrum sensors $(M > 1)$ are assumed to employ their detectors individually to identify the presence of the same signal source and then to send their results to the fusion center as depicted in Figure 1. The spectrum sensors and fusion center are all assumed to be fixed, whereas the signal source –when it is actively transmitting– is assumed to be mobile with an average speed value of $v = 10$m/s and to operate on 2GHz within the same type of propagation environment. The fast–fading channels between the actively transmitting source and the fixed sensors are assumed to have a Rayleigh distributed amplitude with a Doppler spectrum that is of Jakes' type. Spatially correlated log–normal shadowing is applied to the signal with $\sigma_G = \{4.3, 7.5\}$dB and $d_\rho = \{5.75, 350\}$m for urban and suburban environments, respectively, as reported in [33]. These two particular environments are selected intentionally because performance of the proposed method needs to be examined for physical environments which exhibit drastic differences in their propagation characteristics. The sampling frequency of the receiver is assumed to be fixed at 20KHz in order to satisfy the condition $f_D \ll f_s$. The effective averaging time of the low–pass integrate–and–dump filter is set to 0.05ms, whereas the total sensing time is set to 1ms. The impact of path loss is neglected under hypothesis H_1 driven by the fact that sensors generate their results in a very short period of time which leads to a very short displacement that the mobile traverses within that period. In light of this, spectrum sensors are assumed to observe the same SNR, which is selected to be 3.5dB, reflecting a relatively strong presence when the source is actively transmitting.

In order to see the beneficial impact of cooperation for the given settings, Figure 3 should be examined first. It is clear in the figure that the increase in the number of spectrum sensors contributing to decision making mechanism improves the overall system performance drastically, as expected. As will be shown subsequently, this behavior is maintained by the proposed method regardless of the type of detector employed

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in sensors or of the propagation environment considered.

The performance of the proposed method when a drift in concepts takes place, such as a change in propagation environment from suburban to urban, needs to be investigated as well. For this purpose, the proposed method is tested for an urban environment scenario and results are presented in Figure 3 along with the suburban environment results, for the sake of comparison. The overall performance of the proposed method exhibits the same behavior in the urban scenario as it does in the suburban scenario. However, it is important to note that the proposed method performs slightly better in the suburban scenario than in the urban scenario. This stems from the fact that second–order statistical detectors employed in sensors can take advantage of the large decorrelation distance that shapes the shadowing process and therefore performs better. In this regard, although the suburban environment has a greater standard deviation, its larger decorrelation distance dominates and yields better results.

For comparison purposes, the performance of the proposed method can be examined by replacing each second–order detector in sensors with the conventional energy detector while keeping all of the remaining settings intact. Results for such a scenario are also shown in Figure 3. As can be seen from the plots, the energy detector–based adaptive fusion mechanism performs worse compared to the second–order detector–based adaptive fusion framework. This is because the second–order detectors are able to extract the correlation information from the received signal even though it undergoes both fast– and slow–fading. On the other hand, the energy detector relies solely on the power statistics which can be severely degraded by shadowing and deep fast–fading scenarios; therefore, it yields a weaker performance. Here, it is important to state that the second–order detector used in sensors can be transformed into the conventional energy detector by deactivating several blocks, which can be considered within the software–defined radio (SDR) concept.

Although it is not required by the online adaptive data fusion framework, in accordance with the POCS theory, Algorithm 1 is shown to converge by exhibiting a decrease in the average error values over time in Figure 4. Note in Figure 4 that the channel conditions between the source and the sensors are kept the same throughout the convergence period for illustrative purposes. The average error is evaluated for P_f values greater than 0.1. The convergence rate can be adjusted by changing the relaxation parameter μ in (32). It is important to state that for smaller values of P_f, which correspond to more demanding cases, the convergence rate will be smaller accordingly resulting in longer durations to achieve similar average error values.

As stated earlier, energy detector is a special case of the second–order detector. Also, theoretically speaking, output of the energy detector is unbounded, whereas output of the second–order detector, namely V_U, is bounded. This implies that employing energy detection at sensors will challenge the decision fusion mechanism and provide a practical upper bound in terms of (P_f, P_d) pair, as shown in Figure 3. Furthermore, as can be observed in (5), many critical practical concerns such as imperfect carrier and phase recovery are disregarded in the analysis. Hence, the robustness of the proposed method is tested with a more practical scenario which includes the physical implementation of the system model [28, 29]. In [28, 29], the robustness of the proposed method is tested including the following three aspects: First of all, real wireless signals are generated and captured over the air. Second, the fusion center in the experimental setup is exposed to receiver impairments such as inherent low–pass filtering, imperfect carrier and phase recovery. Third, based on the results plotted in Figure 3, energy detector is employed at the sensors in order to challenge the overall performance of the method proposed further. Results in [28, 29] show that the decision fusion mechanism is robust under practical conditions including the impact of real wireless propagation environment and receiver impairments. As reported and discussed in [28, 29], correlated input to the ADF scheme has a positive impact on the performance of cooperative spectrum sensing by yielding lower error values which in turn results in fast convergence of weights.

V. Conclusion

An online adaptive cooperative spectrum sensing scheme based on the POCS theory is proposed in order to maintain the performance and the reliability of sensing under dynamically changing channel and environmental conditions such as correlated shadowing and fast-fading. Each collaborating sensor performs a second-order analysis on the received signal which is assumed to be transmitted by a single mobile source. The contributions of this study are three–fold. First, main propagation mechanisms including fast- and slow-fading phenomena that affect mobile radio channel are incorporated into spectrum sensing. Second, the received signal by sensors are processed with second-order detectors which provide the fusion center with constant output. Finally, an online adaptive data fusion scheme to be deployed in a cooperative spectrum sensing framework is introduced.

Results show that the proposed method improves the performance of the second–order detector with the aid of cooperation. Also, comparative analysis reveals that the adaptive fusion mechanism supported by second–order detector output exhibits superior performance over the conventional energy–based spectrum detectors.

VI. Acknowledgment

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Appendix
Proof of Proposition 1

Proof: Under the hypothesis H_0 note that $T \to \infty$ and $T_A = 0$ imply that there is infinite support with no low–pass filtering; therefore, in (7), $N_F(t)$ immediately degenerates to $n(t)$. In that case, input to the logarithm operator has a chi–square distribution with two degrees of freedom $(\chi^2(2))$.

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However, since input to the logarithm operator is generated by the squaring device, one can take advantage of the equality:

$$\ln\left(|n(t)|^2\right) = 2\ln\left(|n(t)|\right).$$

This way, the input–output relationship for the logarithm operator can be expressed in terms of the Rayleigh distribution, which provides analytical tractability in the subsequent steps. Because $\mathcal{Y}(t) = |n(t)|$ is Rayleigh distributed, its probability density function (PDF) is given by:

$$p_{\mathcal{Y}}(y) = \begin{cases} \frac{y}{\alpha} e^{\left(-\frac{y^2}{2\alpha}\right)}, & 0 \leq y \\ 0, & y < 0 \end{cases} \tag{33}$$

where α is the mode parameter of the distribution satisfying $\mu_{\mathcal{Y}} = \alpha\sqrt{\frac{\pi}{2}}$ with $\mu_{\mathcal{Y}}$ being the mean of the PDF of $\mathcal{Y}(t)$ and $\alpha = \sqrt{\sigma_N^2/2}$. Therefore, output of the logarithm, say $\mathfrak{X}(t)$, forms a time series that is composed of $\log$-Rayleigh distributed values:

$$\mathfrak{X}(t) = \ln\left(\mathcal{Y}(t)\right) = \ln\left(|n(t)|\right), \tag{34}$$

with the following PDF:

$$p_{\mathfrak{X}}(\mathfrak{x}) = \frac{e^{2\mathfrak{x}}}{\alpha}\exp\left(-\frac{e^{2\mathfrak{x}}}{2\alpha}\right) \tag{35}$$

for all $\mathfrak{x} \in \mathbb{R}$. Since the output of the correlator is difficult to express in closed–form when the input is solely noise, let:

$$r'(t) = \lim_{A\to 0} A\cos\left(2\pi f_A t + \phi_A\right) + n(t), \tag{36}$$

where A, f_A, and ϕ_A are some arbitrary amplitude, frequency, and phase values, respectively. Note that (36) is equivalent to the hypothesis H_0 in the limiting sense. If $r'(t)$ follows through the steps (7)–(10), then output of the correlator is given by [46]:

$$\begin{aligned}\Psi(\tau) = &\sum_{\substack{i=1 \\ (i+l\text{ even})}}^{\infty} \varphi_N^k(\tau)\sum_{l=1}^{i}\binom{(i+l)/2-1}{(i-l)/2}\Upsilon^m \\ &\times {}_1F_1^2\left((i+l)/2; l+1; -\Upsilon\right)/l!l\left(i+l\right) \\ &+\frac{1}{4}\sum_{i=1}^{\infty} {}_1F_1^2\left(i;1;-\Upsilon\right)\varphi_N^{2k}(\tau) \\ &+\left(\ln\left(A\right) + \frac{1}{2}\mathfrak{E}_1(\Upsilon)\right)^2,\end{aligned} \tag{37}$$

where $\varphi_N(\cdot)$ is the normalized autocorrelation estimates of the quadrature components ($i.e.$, $n_Q(\cdot)$) of $n(t)$, Υ is the SNR and defined to be $\Upsilon \triangleq A^2/\left(2\sigma_N^2\right)$, ${}_1F_1^2\left(\cdot;\cdot;\cdot\right)$ is the confluent hypergeometric function, and $\mathfrak{E}_1(\cdot)$ is the exponential integral [47]. Since the purpose is to obtain the characteristics of the noise–only process, one can consider (37) by expanding $\mathfrak{E}_1(\cdot)$ into power series for $A \to 0$ (or equivalently for $\Upsilon \to 0$). This allows one to see that (37) is dominated by the cross–noise terms as Υ diminishes, and can be expressed after some manipulations as:

$$R_{\mathfrak{X}}(\tau) \triangleq \lim_{\Upsilon\to 0}\Psi(\tau) \cong \frac{1}{4}\sum_{i=1}^{\infty}\frac{\varphi_N^{2k}(\tau)}{i^2} + \Phi(\sigma_N), \tag{38}$$

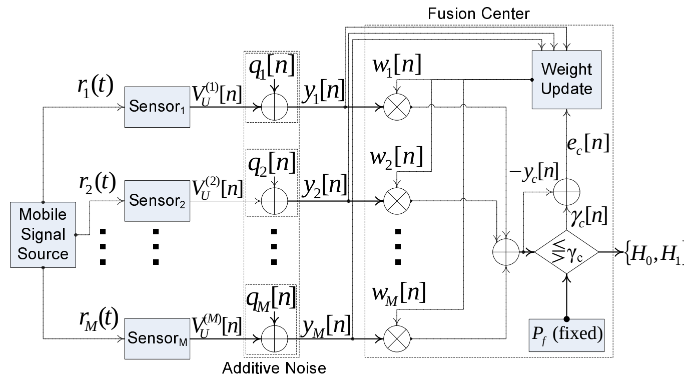

Fig. 1. Block diagram for the proposed method and for the second–order detector embedded in spectrum sensors.

where $\Phi(\sigma_N)$ represents a constant that depends on the noise variance σ_N^2. Since $\Phi(\sigma_N)$ is a constant, one can readily calculate the variance of $\mathfrak{X}(t)$ by setting $\tau = 0$ and ignoring $\Phi(\sigma_N)$ as:

$$\sigma_{\mathfrak{X}}^2 = \frac{1}{4}\sum_{i=1}^{\infty}\frac{1}{i^2} = \frac{\pi^2}{24}. \tag{39}$$

Recalling that $\mathfrak{X}(t)$ is a non–zero mean process ($i.e.$, $\mu_{\mathfrak{X}} \neq 0$) due to the non–linear transformation applied $\Phi(\sigma_N) = \mu_{\mathfrak{X}}^2$ holds in (38), since $R_{\mathfrak{A}}(\tau) = E\left\{\mathfrak{A}(t)\mathfrak{A}^*(t+\tau)\right\}$ and $R_{\mathfrak{A}}(0) = \sigma_{\mathfrak{A}}^2 + \mu_{\mathfrak{A}}^2$ for any stationary stochastic process $\mathfrak{A}(t)$ with $\mu_{\mathfrak{A}} \neq 0$.

Then by assuming σ_N to be unity for the sake of simplicity, $\mu_{\mathfrak{X}}$ can be calculated via (33) and (35)–(38) as:

$$\begin{aligned}\mu_{\mathfrak{X}} &= \ln\left(\alpha\right) + \frac{\ln\left(2\right) - \gamma}{2} \\ &= \ln\left(\frac{1}{\sqrt{2}}\right) + \frac{\ln\left(2\right)-\gamma}{2} = -\frac{\gamma}{2},\end{aligned} \tag{40}$$

where γ is Euler–Mascheroni (or sometimes referred to solely as Euler's) constant and given by $\gamma = -\int_0^{\infty}\ln\left(u\right)e^{-u}du$.[4] In (38), it is clear that at larger delays (lags) τ, the autocorrelation estimates exhibit an asymptotic behavior and converge $\Phi(\sigma_N)$, which is a function of noise variance. However, it is desired that the method proposed is independent of noise variance σ_N^2. Thus, normalizing the autocorrelation estimates with the signal power ($i.e.,$ with the value at $\tau = 0$) will yield the following constant:[5]

$$\Phi' = \frac{\Phi(\sigma_N)}{R_{\mathfrak{X}}(0)} = \frac{\mu_{\mathfrak{X}}^2}{\mu_{\mathfrak{X}}^2 + \sigma_{\mathfrak{X}}^2}. \tag{41}$$

Finally, if (40) is placed in (41), the following is obtained:

$$\Phi' = \frac{\gamma^2}{\gamma^2 + \pi^2/6} \tag{42}$$

which concludes the proof.

■

[4]The first five digits of γ in decimal form are $\gamma \cong 0.57721\ldots$

[5]When σ_N is unity: $\Phi' = \frac{\gamma^2}{\gamma^2+\pi^2/6} \cong 0.16843\ldots$ for the first five significant digits in decimal.

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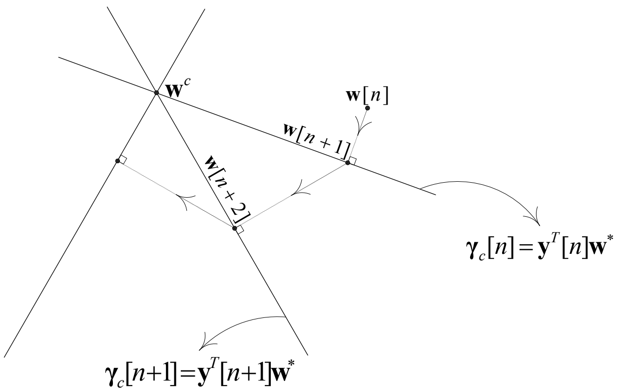

Fig. 2. Geometric interpretation: Weight vectors corresponding to correlator estimates at each time step are updated as to satisfy the hyperplane equations defined by the test threshold $\gamma_c[n]$ and the output of the broadcast fusion channel, $\mathbf{y}[n]$. Here, the lines represent hyperplanes in $\mathbb{R}^M$. If the channel and environmental conditions are kept fixed, iterated weights converge at the intersection of hyperplanes, $\mathbf{w^c}$, as discussed in [42].

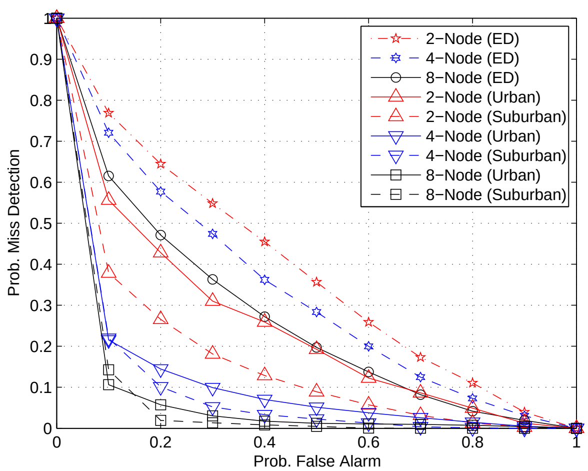

Fig. 3. The performance comparison for the adaptive fusion decision mechanism for different number of spectrum sensors which employ the conventional energy detector (labeled with "(ED)" in the plots) and second–order detector in different scenarios.

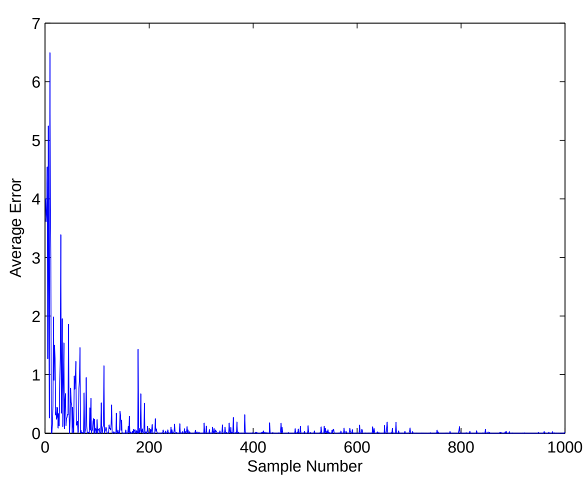

Fig. 4. Convergence of the Algorithm 1 for $0.1 < P_f$.

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