

FILTER BANKS WITH IN BAND CONTROLLED ALIASING APPLIED TO DECOMPOSITION/RECONSTRUCTION OF ECG SIGNALS

F. Cruz-Roldán; F. López-Ferreras; S. Maldonado-Bascón; R. Jiménez-Martínez

Dep. de Teoría de la Señal y Comunicaciones; Escuela Politécnica (U. A.)

e-mail: fernando.cruz@alcala.es; fcruz@euitt.upm.es

28871 Alcalá de Henares, SPAIN

ABSTRACT

This paper presents the realization of a pseudo-Quadrature Mirror Filter (QMF) cosine-modulated filter bank with low amplitude distortion at the frequencies around $\omega=0$ or $\omega=\pi$, and high but controlled aliasing in specific bands. We obtain the analysis and synthesis filters as Spectral Factorization Approach, but the prototype is different because is a symmetric linear-phase filter. This filter bank is applied to the decomposition and reconstruction of an ECG signal. We propose some parameters appropriate to measure amplitude distortion and aliasing, and the reconstruction error in the output signal.

1. M-CHANNEL PARALLEL FILTER BANK

An M -channel maximally-decimated parallel filter bank [1,2,3,4] (figure 1) consists of both analysis and synthesis filters. The analysis bank decompose an input signal $x[n]$ into its subband components in order to use them in many applications as subband coding of audio and images. The synthesis bank reconstructs the M subband signals using filters $F_k(z)$.

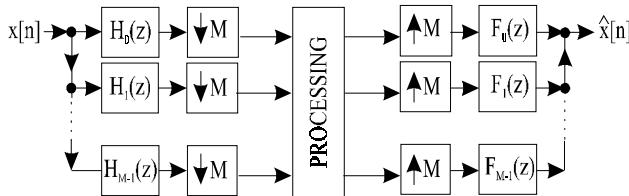


Figure 1: M -channel maximally decimated filter bank.

The reconstructed signal $\hat{x}[n]$ can be written as shown in (1), where $W_M = e^{-j\frac{2\pi}{M}}$:

$$\hat{x}[n] = \frac{1}{M} \sum_{\ell=0}^{M-1} t_{\ell}[n] * (x[n] \cdot W_M^{-\ell n}) \quad (1)$$

The Z transform of the expression before is:

$$\hat{X}(z) = \frac{1}{M} \sum_{\ell=0}^{M-1} T_{\ell}(z) X(z W_M^{\ell}) \quad (2)$$

We can rewrite expression (2) in a more convenient form (expression (3)) in order to see separately amplitude and phase distortions, and aliasing [5].

$$\hat{X}(z) = \frac{1}{M} X(z) T_0(z) + \frac{1}{M} \sum_{\ell=1}^{M-1} T_{\ell}(z) X(z W_M^{\ell}) \quad (3)$$

The overall distortion transfer function $T_0(z)$ is:

$$T_0(z) = \sum_{k=0}^{M-1} F_k(z) H_k(z) \quad (4)$$

whereas $T_{\ell}(z)$ $1 \leq \ell \leq (M-1)$ represents the $(M-1)$ aliasing transfer functions corresponding to $X(z W_M^{\ell})$:

$$T_{\ell}(z) = \sum_{k=0}^{M-1} F_k(z) H_k(z W_M^{\ell}) \quad (5)$$

When the filter bank has the perfect reconstruction (PR) property, it satisfies restrictive design conditions to cancel amplitude and phase distortions, and aliasing. In pseudo-QMF banks, the conditions are not so restrictive,

and it is only necessary that the overall distortion transfer function $T_0(z)$ must be linear-phase and

$$\begin{cases} 1 - \delta_1 \leq |T_0(e^{j\omega})| \leq 1 + \delta_2 \\ |T_\ell(e^{j\omega})| \leq \delta_3 \end{cases} \quad 1 \leq \ell \leq M-1 \quad (6)$$

where δ_1 , δ_2 , and δ_3 are very small numbers.

Furthermore, when the original signal localizes its energy in a determined frequency region, the amplitude distortion must be as low as possible in this region but $|T_\ell(e^{j\omega})|$ can increase its value in the rest of the frequencies. So, we can define a specification for $|T_\ell(e^{j\omega})|$ where we find frequency regions with very low aliasing and other bands where aliasing is controlled.

In this paper we propose an approach to pseudo-QMF cosine-modulated filter banks with a low level of amplitude distortion in low or high frequencies. Although this approach increases the aliasing in localized regions, it can be controlled. We apply this filter bank to decomposition and reconstruction of an ECG signal.

2. THE ANALYSIS AND SYNTHESIS FILTERS

In a pseudo-QMF bank we must design the prototype filter $P(z)$ using a weighted objective function that minimizes the stopband attenuation and the overall amplitude distortion [1,2,4]. In this approach, the way of obtaining the analysis and synthesis filters is different. Let $P(z) = \sum_{n=0}^{N-1} p[n] \cdot z^{-n}$ be a symmetric linear-phase prototype filter with real coefficients. As in the spectral factorization approach [6], we define the following time domain relations:

$$s_k[n] = 2p[n] \cos\left(\left(k + \frac{1}{2}\right) \frac{\pi}{M} n + \phi_k\right) \quad (7)$$

The analysis filters $h_k[n]$ are obtained by alternating $s_k[n]$ and its 'flipped' versions $s_k[N-1-n]$ as given below

$$h_k[n] = \begin{cases} s_k[n] & k \text{ even} \\ s_k[N-1-n] & k \text{ odd} \end{cases} \quad (8)$$

$$\begin{cases} 0 \leq n \leq N-1 \\ 0 \leq k \leq M-1 \end{cases}$$

If aliasing has been canceled, the filter bank will be free from phase distortion provided that the synthesis filters are chosen according to

$$f_k[n] = h_k[N-1-n] \quad (9)$$

$$\begin{cases} 0 \leq n \leq N-1 \\ 0 \leq k \leq M-1 \end{cases}$$

If the Z transform is calculated, the analysis $H_k(z)$ and synthesis $F_k(z)$ filters are

$$H_k(z) = \begin{cases} S_k(z) & k \text{ even} \\ z^{-(N-1)} \tilde{S}_k(z) & k \text{ odd} \end{cases} \quad (10)$$

$$F_k(z) = z^{-(N-1)} \tilde{H}_k(z) \quad 0 \leq k \leq (M-1) \quad (11)$$

where $\tilde{S}_k(z) = S_k(z^{-1})$.

3. AMPLITUDE DISTORTION AND ALIASING

In order to evaluate the quality of the filter bank we use two measures: the amplitude distortion in the overall distortion transfer function $|T_0(e^{j\omega})|$ and the aliasing in aliasing transfer functions.

If the prototype filter is a symmetric linear-phase filter, pseudo-QMF designs show the overall distortion function transfer as

$$\begin{aligned} T_0(z) = & \sum_{k=0}^{2M-1} P^2(zW_{2M}^{(k+1/2)}) \cdot W_{2M}^{(k+1/2)(N-1)} + \\ & P(zW_{2M}^{1/2}) \cdot P(zW_{2M}^{-1/2}) \cdot 2 \cdot \cos\left(\frac{1}{2} \cdot \frac{\pi}{M} \cdot (N-1) + 2\phi_0\right) + \\ & P(zW_{2M}^{(M-1/2)}) \cdot P(zW_{2M}^{-(M-1/2)}) \cdot 2 \cdot \cos\left(\left(M-1 + \frac{1}{2}\right) \cdot \frac{\pi}{M} \cdot (N-1) + 2\phi_{M-1}\right) \end{aligned} \quad (12)$$

Figures 2 and 3 show the magnitude response of $T_0(z)$ for four-channel pseudo-QMF cosine-modulated filter banks designed with two different prototypes. The filter design technique is eigenfilter approach [7], but the orders are 120 (Figure 2) and 122 (Figure 3). We have measured the maximum peak to peak amplitude distortion by the expression (13) in these functions. The results are given in Table 1.

$$R_{pp} = 20 \log \left(\left| T_0(e^{j\omega}) \right|_{MAX} - \left| T_0(e^{j\omega}) \right|_{MIN} \right) \quad (13)$$

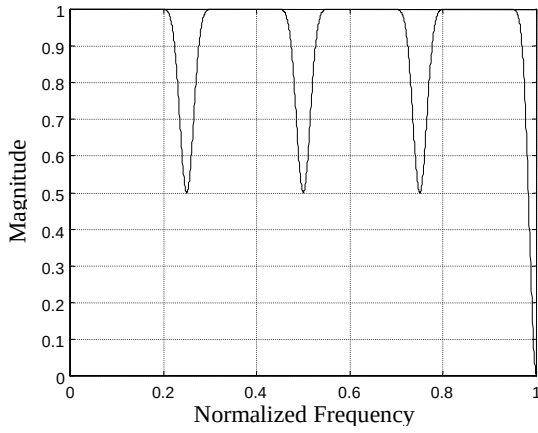


Figure 2: magnitude response of $T_0(e^{j\omega})$ ($N=120$).

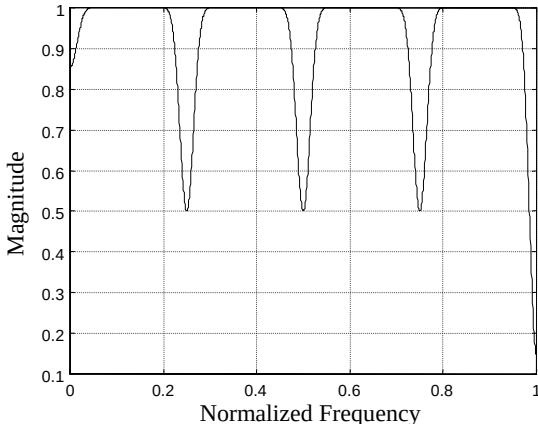


Figure 3: magnitude response of $T_0(e^{j\omega})$ ($N=122$).

Regarding aliasing, when the order of the prototype filter is a multiple of M , if we choose the angles ϕ_k as

$$\phi_{k+1} = \pm(2i+1)\frac{\pi}{2} - \phi_k \quad 0 \leq k \leq M-2 \quad (14)$$

where i is an integer, we will ensure that all the significant aliasing terms are canceled [6].

The aliasing may increase if the prototype order is not a multiple of M , but it can be controlled in amplitude and wide. It occurs in certain frequency regions and it is not really important when the energy of the input signal is negligible at these frequencies.

In order to measure the total aliasing it is defined the aliasing function $T_{al}(e^{j\omega})$ as shows (15), which is a measure of every aliasing component contribution [2][3].

$$T_{al}(e^{j\omega}) = \sqrt{\sum_{\ell=1}^{M-1} \left| \frac{1}{M} T_{\ell}(e^{j\omega}) \right|^2} \quad (15)$$

The quantity E_a , defined as the maximum value of aliasing function $T_{al}(e^{j\omega})$, is useful to measure aliasing because it is the worst possible *peak aliasing distortion* [2].

Figures 4 and 5 show the magnitude response of the aliasing function $T_{al}(z)$ for the four-channel pseudo-QMF cosine-modulated filter banks given before. In Figure 5 can be seen frequency regions around $\omega=0.25\pi$ and $\omega=0.75\pi$ where there is a high but controlled aliasing level. The highest aliasing level is similar to the maximum peak to peak amplitude distortion at these frequencies. Moreover, the aliasing level is smaller than -100 dB outside the controlled band, similar to stopband attenuation of $P(z)$.

In our examples we have chosen the angles as

$$\phi_k = \begin{cases} \pi/2 & k \text{ even} \\ 0 & k \text{ odd} \end{cases} \quad (16)$$

4. THE OUTPUT SIGNAL AND THE RECONSTRUCTION ERROR

The quality of the reconstructed signal can be measured employing the percent root-mean-square difference (PRD) or the signal to noise

ratio (SNR) as given (17) and (18) respectively, where $x[n]$ and $\hat{x}[n]$ are the original and reconstructed signals. Reconstructing with low PRD or high SNR does not necessarily mean clinical acceptance, but they are accepted error measures [1].

$$PRD = \sqrt{\frac{\sum (x[n] - \hat{x}[n])^2}{\sum (x[n])^2}} \cdot 100 \quad (17)$$

$$SNR = 10 \log \left(\frac{\sum (x[n])^2}{\sum (\hat{x}[n] - x[n])^2} \right) \quad (18)$$

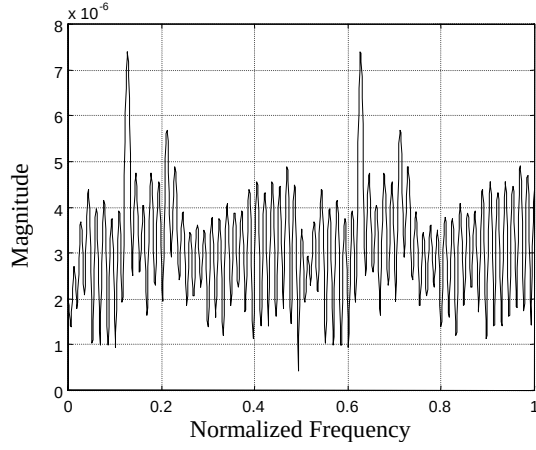


Figure 4: magnitude response of $T_{al}(e^{j\omega})$ ($N=120$).

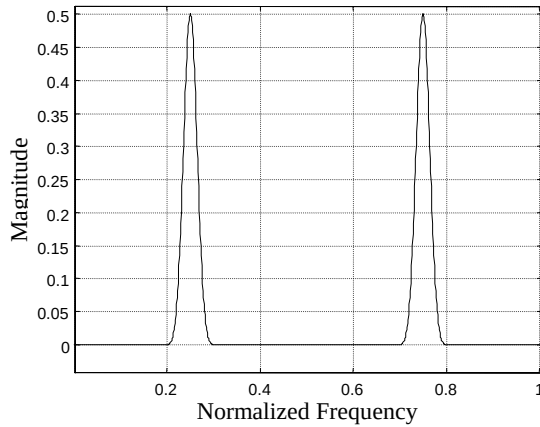


Figure 5: magnitude response of $T_{al}(e^{j\omega})$ ($N=122$).

Figures 6 and 7 show the original signal (before filter bank) and the reconstructed signal (after filter bank), and the results are given in Table 1.

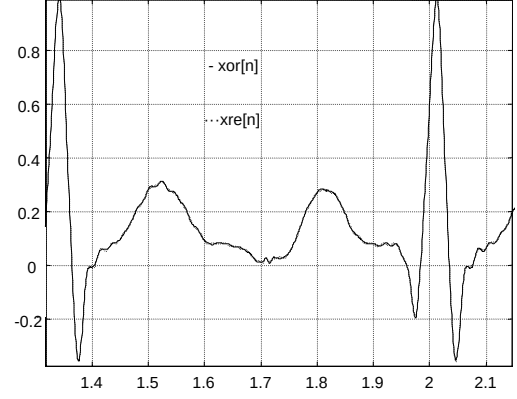


Figure 6: detail of original and reconstructed signals ($N=120$).

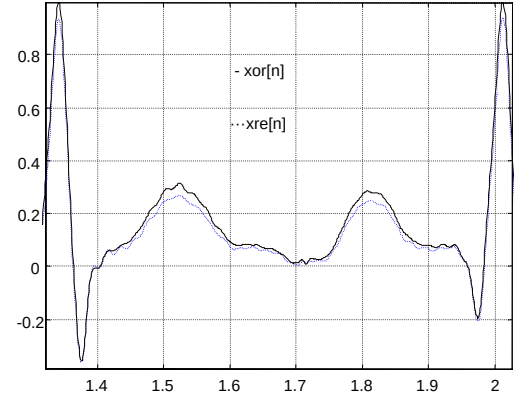


Figure 7: detail of original and reconstructed signals ($N=122$).

Table 1: results on several pseudo-QMF filter banks

Order	PRD	SNR	R_{pp}	E_a (dB)
120	1.8195	34.8011	65.2954	-102.623
122	10.7794	19.3481	16.6587	-6.0204

5. SUMMARY

Our work indicates that we can use a symmetric linear-phase filter as prototype of pseudo-QMF cosine-modulated filter bank in order to decompose the ECG signal in subbands and reconstruct it using these subbands, because the filter bank shows an amplitude distortion lower than other approaches. The resulting filter bank has low amplitude distortion at the frequencies around $\omega=0$, where the ECG signal localizes almost all its energy, and therefore, the value of *PRD* decreases. The aliasing increases if the prototype order is not a multiple of *M*, the number of channels. It may occur in certain frequency regions, but it can be controlled in amplitude and wide.

6. REFERENCES

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